Non-iterative imaging of inhomogeneous cold atom clouds using phase retrieval from a single diffraction measurement

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Abstract: We demonstrate a new imaging technique for cold atom clouds based on phase retrieval from a single diffraction measurement. Most single-shot diffractive imaging methods for cold atoms assume a monomorphic object to extract the column density. The method described here allows quantitative imaging of an inhomogeneous cloud, enabling recovery of either the atomic density or the refractive index, provided the other is known. Using ideas borrowed from density functional theory, we calculate the approximate paraxial diffracted intensity derivative from the measured diffracted intensity distribution and use it to solve the Transport of Intensity Equation (TIE) for the phase of the wave at the detector plane. Back-propagation to the object plane yields the object exit surface wave and then provides a quantitative measurement of either the atomic column density or refractive index. Images of homogeneous clouds showed good qualitative agreement with conventional techniques. An inhomogeneous cloud was created using a cascade electromagnetically induced transparency scheme and images of both phase and amplitude parts of refractive index across the cloud were separately retrieved, showing good agreement with theoretical results.

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References and links
1. Introduction

Measurement of the spatial distribution of cold atoms has become increasingly important with the development of ultracold atom trapping and experiments in a diverse range of applications. Atomic coherence processes, such as generation of entangled states in the Rydberg blockade regime [2, 3], Electromagnetically Induced Transparency (EIT) [4] and slow light [5] are all typically investigated using simple probe beam techniques which do not provide spatially resolved information. More recently, transfer of optical information between spatially separated...
Bose-Einstein condensates [6] has provided further motivation for investigation of more complicated atomic coherence phenomena in cold gases. Imaging of the atomic distribution can potentially provide increased information on the underlying processes.

More directly, we are interested in imaging of cold atoms as a means of controlling electron bunch shapes generated from Ultracold Plasma (UCP). UCP is produced by photoionization of a cold atom cloud, resulting in cold ions and electrons [7]. Several laser fields, each with a spatially varying intensity profile, will interact with the cold atoms to produce the UCP. The effect of these fields on the refractive index of the cloud will result in an inhomogeneous atom cloud. Imaging of the cloud can provide feedback to control the final electron density distribution and ultimately the electron brightness, via changes in the ionization laser intensity profiles. Electron bunches extracted from an UCP are potentially much brighter than conventional thermal sources [8] and in the long-term offer the possibility of sufficient brightness for single-shot diffractive imaging of bio-molecules.

Conventional imaging techniques for cold atoms involve absorption of a probe laser tuned to the atomic resonance [9]. An in-focus absorption “shadow” image is recorded, and knowledge of the atomic refractive index is used to retrieve a spatial map of atomic column density. While experimentally simple, such absorption imaging is very sensitive to experimental parameters such as the frequency stability and detuning of the illuminating laser beam and defocus of the imaging optics. For example, an inadvertent half-natural-line-width detuning from resonance results in a factor of three error in the resulting atomic density measurement. Furthermore, for high-density clouds such as Bose-Einstein Condensates (BECs), saturation of the absorption signal can result in an inaccurate quantitative measurement.

Phase imaging techniques, such as Zernike phase-contrast imaging, offer an alternative solution which greatly reduces this sensitivity, but this comes at the cost of increased experimental complexity and such techniques are quantitative only for a limited range of phase shifts. Diffraction Contrast Imaging (DCI), [10, 11, 12] offers an experimentally simpler method of quantitative phase imaging. DCI computationally inverts off-resonant diffraction data to provide a map of atomic density. The inversion relies on the assumption of an homogeneous object, that is, one having a refractive index which varies only with the density, without any inherent spatial dependence. Processes involving atomic coherence typically rely on many laser fields interacting with the cold atoms, each of which have spatially varying intensity profiles. The effect of these laser fields on the atomic refractive index results in an inhomogeneous atom cloud. Accordingly, such objects of interest cannot be imaged using DCI, even with knowledge of the spatial variation of atomic refractive index.

To enable imaging of inhomogeneous objects, we use techniques developed in the context of x-ray phase contrast imaging and coherent diffractive imaging [1], adapted to imaging of cold atoms. Using a non-iterative phase retrieval method, we quantitatively extract the atomic column density of an atom cloud from a single plane of diffraction data, given knowledge of the atomic refractive index. In addition, we use a previously measured column density to extract the complex refractive index of an inhomogeneous cloud. The Single-plane, Curved-beam Phase Imaging method (SCPI) retains the experimental advantages of diffractive imaging techniques like DCI, while avoiding the restrictive homogeneous object requirement.

2. Method

A conceptual diagram of the imaging technique is shown in Fig. 1. The atom cloud is defined in terms of the atomic column density, \( \rho(r) \), which is given by the integral of the atom number density \( N(r,z) \) along the optical path \( z \):

\[
\rho(r) = \int_{-\infty}^{0} N(r,z) \, dz
\]
Curved incident wavefront

Fig. 1. Conceptual arrangement for Single-plane Curved-beam Phase Imaging. A phase curved Gaussian beam is incident on the atom cloud at plane $z_1$, diffracts and propagates a distance $Z$ to a CCD detector at plane $z_2$.

where $z$ is in the direction of propagation and $r$ is the transverse coordinate. For a thin object, the exit surface wave $\psi(r, z_1)$ is related to the incident field $\psi_0(r, z_1)$ via the transmission function, $T(r)$:

$$\psi(r, z_1) = \psi_0(r, z_1) T(r). \quad (2)$$

The transmission function can be written as:

$$T(r) = \exp \left[ ik \{ \alpha(r) + i \beta(r) \} \rho(r) \right] \quad (3)$$

where $\alpha(r)$ and $\beta(r)$ are the spatially varying real and imaginary parts of the complex atomic refractive index, $k = 2\pi/\lambda$ and $\lambda$ is the wavelength of the probe laser beam. Hence the atomic column density is given by:

$$\rho(r) = \frac{1}{k} \left\{ \ln |T(r)|^2 \right\}^{\frac{1}{2}} \frac{\alpha^2(r) + \beta^2(r)}{\alpha^2(r) + \beta^2(r)} \quad (4)$$

Alternatively, if the density is known and the refractive index variation is desired, rearranging Eq. (3) gives an expression for the complex refractive index in terms of the known column density, $\rho(r)$:

$$\alpha(r) + i \beta(r) = -\frac{1}{ik \rho(r)} \ln \left( \frac{\psi(r, z_1)}{\psi_0(r, z_1)} \right). \quad (5)$$

We measure the intensity at the detector plane, $|\psi(r_2, z_2)|^2$. The central section of the diffraction data is essentially a point projection hologram, while high-angle scatter provides additional information. To retrieve the exit surface wave, $\Psi(r_1, z_1)$, we also require the phase of the wave at the detector, which we can obtain using the Transport of Intensity Equation (TIE) [13]:

$$\nabla_\perp \cdot I(r, z) \nabla_\perp \Phi(r, z) = -k \frac{\partial I(r, z)}{\partial z}. \quad (6)$$

The TIE is a continuity equation which relates the phase of a wave, $\Phi(r, z)$, to its intensity, $I(r, z)$, and longitudinal intensity derivative, $\partial I/\partial z(r, z)$, using the condition of energy conservation. The measured diffraction pattern is the required intensity input. Conventionally, the
derivative term is found by taking two diffraction images separated by some small propagation distance, then approximating the derivative using finite difference methods [15]. Solving the TIE provides the phase of the wave at the detector plane, and back-propagation yields the complex wave-field in the sample plane [14, 16]. The key to the approach described by Quiney et al. [1] is to use a priori information about the unperturbed imaging beam, to calculate an approximate intensity derivative from a single diffraction image. Adaptation of this method to imaging of cold atom clouds provides the advantage that unlike other methods which solve the TIE using a single diffraction image [17], this approach does not assume a homogeneous object.

The calculation of the approximate longitudinal derivative follows the method of Quiney et al. [1]. For an axially aligned Gaussian beam (as in Fig. 1), we describe the unperturbed wave at the detector plane as \( \psi_0(r, z_2) = \exp(-\zeta r^2) \). The real part of \( \zeta \) is related to the 1\( e \) radius of the beam, \( r_0 \), by \( \text{Re}(\zeta) = 1/r_0^2 \). As shown by Quiney et al. [1], the functional relation between the intensity of the unperturbed wave at the detector and its longitudinal derivative is given by \( H[I_0] \):

\[
H[I_0] \equiv \frac{\partial I_0}{\partial z} = -\frac{\zeta}{Z} \int \int w^2 g_0(w) \exp\left(\frac{2\pi i w \cdot r}{\lambda Z}\right) dw
\]  

(7)

where \( Z = z_2 - z_1 \) and \( g_0(w) \) is the autocorrelation of \( \psi_0(r_1, z_1) \exp(i\pi r_1^2/\lambda Z) \). The autocorrelation \( g_0(w) \) is the inverse Fourier transform of the unperturbed intensity \( I_0(r_2, z_2) \), and \( w \) is the transverse coordinate of the autocorrelation. The central approximation of the method is to assume that the functional form of Eq. (7) remains unchanged for the perturbed beam, i.e.

\[
\frac{\partial I}{\partial z} \approx H[I].
\]

(8)

To state this explicitly, we assume that \( H[I] \) is of the same form as in Eq. (7) so that we can replace the unperturbed intensity measurement used to calculate \( g_0(w) \) in Eq. (7) with the perturbed diffraction intensity measurement. This assumption is essentially perturbative: as long as the absorption and phase shifts caused by the object are small and slowly varying, with no discontinuities in their derivatives, the approximation in Eq. (8) demonstrates good agreement with the exact result. In addition to these requirements, the illuminating beam must have significant curvature at the object plane, typically requiring a Fresnel number of five or greater. Such curvature ensures that the far-field diffraction pattern is essentially a magnified version of the short-distance Fresnel diffraction pattern, so that the TIE can be solved using standard techniques for short propagation distance diffraction [18].

These requirements are all readily accommodated in the context of cold atom imaging. By detuning the probe laser from resonance, we have the advantage of being able to effectively tune the absorption and phase shifts imposed by the object. In addition, cold atom clouds are, by design, smooth Gaussian objects: the phase and absorption perturbations will be slowly varying with no discontinuities or phase vortices [19]. The validity of the approximation as a function of probe detuning is investigated using simulations of atom clouds with typical experimental parameters (section 3).

It should be noted that while we have demonstrated the use of a Gaussian incident beam for experimental simplicity, the technique is more generally applicable. As long as the form of \( \psi_0(r, z) \) is known and the above assumptions are satisfied, an expression equivalent to Eq. (7) can be constructed and used to calculate a derivative for input to the TIE.

 Retrieval of the exit wave phase and therefore the column density of the atom cloud (Eq. (4)) is performed using standard techniques to solve the TIE [18]. The method involves division by the measured intensity distribution, which necessarily falls smoothly to zero at the boundaries.
of the image. To maintain a stable solution in these areas well beyond the extent of the object, we divide by a modified form of the intensity:

\[
I(r_2, z_2) = \begin{cases} 
I(r_2, z_2) & I(r_2, z_2) \geq \varepsilon \\
I(r_2, z_2) + \varepsilon & I(r_2, z_2) < \varepsilon 
\end{cases}
\]  

(9)

where \( \varepsilon \ll 1 \) is a regularization parameter chosen to minimize quantitative errors while maintaining a stable solution [20, 21]. Intensities were normalized to one and \( \varepsilon = 10^{-3} \) for simulations and experiments. Smaller values of \( \varepsilon \) introduced numerical errors in the outer regions of the image, while larger values decreased accuracy.

Propagation of the retrieved wave field back to the object plane is performed using the paraxial Fresnel diffraction formalism (Eq. (10)):

\[
\Psi(r_2, z_2) = \frac{-i}{\lambda Z_{\text{rad}}} \exp \left( \frac{2\pi i Z_{\text{rad}}}{\lambda} \right) \exp \left( \frac{i\pi r_2^2}{\lambda Z_{\text{rad}}} \right) F \left\{ \Psi(r_1, z_1) \exp \left( \frac{i\pi r_1^2}{\lambda Z_{\text{rad}}} \right) \right\}. 
\]  

(10)

\( Z_{\text{rad}} = ZR / (Z + R) \) is the rescaled propagation distance needed for phase-curved illumination according to the Fresnel scaling theorem [21], \( R \) is the radius of wavefront curvature at the object, and \( F \) indicates a Fourier transform. Finally, the recovered wave field at the object plane is rescaled by the geometric transverse magnification factor \( M = (Z + R)/R \).

Having now retrieved the exit surface waves for both the perturbed and unperturbed beams, Eq. (4) provides the desired map of atomic column density, \( \rho(r) \), or if the column density is known, the refractive index can be extracted using Eqs. (2) and (3).

3. Simulations

To test the validity of the assumption made in Eq. (8), a range of simulations were performed using realistic experimental parameters (eg. Fig. 2).

3.1. Homogeneous object simulations

The atomic column density was constructed from two overlapping atom clouds with spherically symmetric Gaussian density profiles, positioned to construct an asymmetric cloud, off-centre from the optical axis. Several different sizes and shapes of cloud were used. The refractive index of the object was calculated as a function of probe beam detuning, \( \Delta \), using optical Bloch equations for a two-level \(^{85}\)Rb atom [10]. The unperturbed incident Gaussian beam was multiplied by the transmission function (Eq. (3)) to yield the exact exit surface wave. Propagation to the detector was performed using Eq. (10). Equations (8) and (7) were then used to construct the approximate and exact derivatives for the perturbed and unperturbed beams. The phase of each wave was retrieved via solution of the TIE, and the waves were back-propagated to the object plane. Eq. (4) provided the retrieved atomic column density for comparison to the input.

As can be seen from Fig. 2(d), the reconstructed column density shows good qualitative agreement with the input. A line profile of the input and retrieved column densities is shown in Fig. 2(e). Although the shape is clear, there is a discrepancy in the magnitude of the reconstructed column density, arising from two sources. Firstly, the approximation made in Eq. (8) assumes that the object introduces only a small, smoothly varying perturbation to the measured intensity of the unperturbed wave. For our Gaussian probe beam, the form of Eq. (7) can be expressed in terms of the second transverse derivative using standard Fourier techniques [22], i.e. \( H[I] \propto \nabla^2 I \). Strong perturbations will introduce additional terms which are not accommodated by that approximation. Secondly, regularization (Eq. (9)) results in a small quantitative change to the recovered phase, even though the shape remains well defined. Propagation back to the object plane magnifies the effects of these two sources of error.
Fig. 2. Simulation of diffraction from an atom cloud when illuminated by a near-resonant (detuned +1.5 natural line widths from resonance) curved Gaussian beam, and reconstruction of the column density. (a) input column density ($10^{13}$ atoms/cm$^2$), (b) simulated normalized diffraction intensity, (c) approximate intensity derivative (see Eq. (8)), (d) reconstructed atomic column density ($10^{13}$ atoms/cm$^2$). (e) Horizontal line profiles of column density through the point of maximum density in (a), for the input, raw output and worst-case corrected output column densities, as outlined in section 3. Green lines indicate retrieved imaginary part of refractive index (solid) and simulation input (dashed) for inhomogeneous simulations (Sec. 3.3). The refractive index outside the cloud has been removed for clarity.

3.2. Correction factor

The difference between the input and retrieved peak column densities provides a quantitative measurement of the magnitude of this discrepancy, for a given perturbation, as defined by the probe detuning and peak column density. Since the detuning is known and the raw output density is measured, this information can be inverted to calculate a correction factor, $\eta$, to partially account for the errors introduced by the approximations outlined above, such that:

$$\rho_{\text{corrected}} = \eta \rho_{\text{raw}}.$$  

$\rho_{\text{corrected}}$ and $\rho_{\text{raw}}$ are the peak column densities of the corrected and raw outputs respectively, and $\eta = \rho_{\text{input}}/\rho_{\text{raw}}$. The correction factor $\eta$ is a constant (calculated from the peak values of the atomic densities), but is applied across the entire array. By performing multiple simulations across a range of detunings from $0\Gamma \leq \Delta \leq 3\Gamma$ and peak column densities between $10^{12} \leq \rho_{\text{input}} \leq 3 \times 10^{13}$ atoms/m$^2$, $\eta$ was calculated for the range of perturbations typically achieved in our experiments. The calculated correction factor corresponding to the measured detuning and raw column density was applied to the raw density output. Since the correction factor is calculated using the peak column densities but applied to the entire array, some discrepancy between the input and corrected densities remains at the outer edges of the cloud. Simulations showed that this difference between input and corrected density was less than 10% for probe detunings greater than $\sim 1.5\Gamma$ and peak column densities within the range investigated here.

When experimental uncertainty of the probe beam detuning was considered, the simulations indicated that beyond $1.5\Gamma$ the sensitivity of the technique to such uncertainty is greatly reduced. For example, at $\Delta = 2.0\Gamma$, an experimental uncertainty of $\Gamma/2$ in the detuning of the probe beam results in an error in corrected peak density of $\sim 30\%$. By comparison, the same
uncertainty would result in a 200% error in reconstructed density when using conventional on-resonance absorption imaging. The red curve in Fig. 2(e) shows the "worst-case" corrected column density profile corresponding to this scenario, where the value of \( \eta \) was calculated using a \( \Gamma/2 \) error in probe beam detuning. When used to correct experimental data, the correction factor increases the quantitative accuracy of the technique while simultaneously indicating the optimum conditions for reducing sensitivity to experimental parameters.

3.3. Inhomogeneous object simulations

Simulations of imaging EIT in cold atom clouds were also performed. These simulations required a realistic model of the target atom cloud, with a spatially varying refractive index consistent with practical implementations of EIT schemes as discussed later (section 5.2). Optical Bloch equations were used to calculate the refractive index for a 3-level cascade EIT system between the \( 5S \rightarrow 5P \rightarrow 5D \) transitions of rubidium-85 [23, 24].

An analytical expression for the steady-state off-diagonal density matrix element \( \rho_{12} \) was obtained, containing over 100 terms. This exact result was used in simulations, but to provide some insight a first-order Taylor expansion was calculated for \( \rho_{12}/\Omega_1 \), which is proportional to the refractive index of the probe beam:

\[
\frac{\rho_{12}}{\Omega_1} \approx \frac{2i\pi (\Gamma_2 + 2i(\Delta_1 + \Delta_2))}{4\pi^2(\Gamma_1 + 2i\Delta_1)(\Gamma_2 + 2i(\Delta_1 + \Delta_2)) + \Omega_2^2}.
\]

(12)

The off-diagonal density matrix element is dependent on the detunings \( (\Delta_1, \Delta_2) \) and Rabi frequencies \( (\Omega_1, \Omega_2) \) of the probe and coupling lasers. The subscripts 1 and 2 indicate the probe \( (5S \rightarrow 5P, 780 \text{ nm}) \) and coupling \( (5P \rightarrow 5D, 776 \text{ nm}) \) laser respectively, and \( \Gamma_1 \) and \( \Gamma_2 \) are the natural line widths of the corresponding transitions.

To simulate imaging of a cloud with EIT-induced spatial structure, the pump Rabi frequency \( \Omega_2 \) was spatially modulated across the image array. A vertically elongated Gaussian was used as the intensity of the pump laser, running through the atom cloud slightly off-centre, as in experiments discussed below. The intensity-dependent refractive index across the cloud was calculated using the exact result for the density matrix element (approximated in Eq. (12)).

The resulting array was then used to create the transmission function for the inhomogeneous cloud. Simulated diffraction data were created and used to solve the TIE, from which the object exit wave and unperturbed wave were recovered. The spatially dependent refractive index was extracted using Eq. (5). Other forms of single plane diffractive imaging are unable to separately extract the complex refractive index from a known column density and diffraction pattern.

The retrieved refractive index across the cloud was compared to the simulation input, as shown for the imaginary part of the refractive index by the green lines in Fig. 2(e). In regions where the column density was large, quantitative recovery of the spatially varying refractive index showed excellent agreement, to within 15% of the input. This is evident in the discrepancy between the solid and dashed green lines in Fig. 2(e). In regions of low atomic density, reconstructions used Tikhonov regularization, to avoid the singularity in Eq. (5) [20].

4. Experiment

Imaging experiments were performed on both homogeneous and inhomogeneous atom clouds, using a rubidium-85 magneto-optical trap (MOT) in \( \sigma^\pm \) configuration [25, 26]. Cooling and trapping were performed on the \( 5S_{1/2} (F=3) \rightarrow 5P_{3/2} (F'=4) \) hyperfine transition at 780 nm with external cavity diode lasers (ECDLs) [27, 28, 29] and a semiconductor tapered amplifier [30]. Frequency stabilization was achieved using saturated absorption spectroscopy [31]. An additional repump laser resonant with the \( F = 2 \rightarrow F' = 3 \) hyperfine transition was co-propagated along one axis of the cooling beams. Atoms were provided using a dispenser source. The peak
density of the cloud could be controlled using a combination of the detuning of the trapping beams, variation of the trap magnetic fields and the current through the dispenser source. An acousto-optic modulator (AOM) was used to switch the imaging beam on and off (for timed exposures) before the beam was guided to the MOT via single-mode polarization maintaining optical fiber.

![2-D and 3-D schematics of the imaging and pump beam paths, and MOT vacuum chamber (not to scale). Waveplates have been omitted for clarity, as have coils in 3D. The pump beam was blocked for homogeneous imaging (section 4.1) and unblocked to create the inhomogeneous atom cloud (section 5.2). BS: Beam splitter; PD: Photodiode.](image)

At the vacuum chamber, the imaging beam was collimated to a $1/e^2$ diameter of 5 mm. An $f = 50$ mm achromatic doublet lens was used to focus the beam 70 mm behind the atom cloud (Fig. 3). The Fresnel number of the beam at the object plane was limited to a maximum of $N_F \sim 5$, due to restricted optical access to the vacuum chamber. A 200 mm lens\textsuperscript{1} allowed the effective defocus distance to be varied easily. Images were acquired with an interline transfer CCD\textsuperscript{2}.

\textsuperscript{1}Nikon micro-Nikkor AF 200 mm f/4
\textsuperscript{2}Apogee Alta U2000 ML 1600 x 1200 pixels
4.1. Homogeneous imaging

For the homogeneous cloud, the imaging laser was detuned a frequency \( \Delta \) from the \( F = 3 \rightarrow F' = 4 \) transition (natural linewidth \( \Gamma = 6 \text{ MHz} \)). The calculated refractive index of a two-level Rb-85 atom was used to extract a quantitative measurement of the atomic column density.

4.2. Inhomogeneous imaging

For imaging of an inhomogeneous atom cloud, an additional laser was used to modify the refractive index of the object. This “pump beam” was used to couple the upper transition in a cascade electromagnetically induced transparency scheme on the 5\( S_{1/2}(F = 3) \rightarrow 5\)P\(_{3/2}(F' = 3) \rightarrow 5\)D\(_{3/2}(F'' = 2, 3, 4) \) transitions [33, 34]. Only a section of the cloud was illuminated with this beam, so that only part of the cloud exhibited EIT behavior. Accordingly, the refractive index of the illuminated section was modified by the presence of the pump beam, while the remainder of the cloud was unaffected, resulting in a spatial modulation of refractive index across the extent of the cloud.

To create the EIT scheme, a second semiconductor tapered amplifier (maximum output power \( \sim 300 \text{ mW} \)) seeded by a 776 nm ECDL provided the high power pump beam on the upper transition. The imaging beam also acted as the probe beam on the lower transition. To illuminate only a section of the MOT, the 776 nm beam was focused through a slit (width \( \sim 500 \mu \text{m} \)) using a cylindrical lens (\( f = 100 \text{ mm} \)). The slit was re-imaged at the MOT using two \( f = 300 \text{ mm} \) lenses, to create a “light sheet” (Fig. 3).

The pump beam was frequency locked to the two step 5\( S_{1/2}(F = 3) \rightarrow 5\)P\(_{3/2}(F' = 4) \rightarrow 5\)D\(_{3/2}(F'' = 5) \) transition [35] then shifted 130 MHz using an AOM, to be on resonance with the 5\( P_{3/2}(F' = 3) \rightarrow 5\)D\(_{3/2}(F'' = 2, 3, 4) \) transitions at the MOT.

For imaging, the probe laser was locked \( +72 \text{ MHz} \) blue of the 5\( S_{1/2}(F = 3) \rightarrow 5\)P\(_{3/2}(F' = 3) \) transition and shifted back to resonance using an AOM, which also allowed switching of the imaging beam for timed exposures. Variation of the lock point allowed detuning of the imaging beam from resonance, while variation of the 776 nm AOM frequency allowed the relative frequency of the EIT resonances to be independently adjusted. The imaging beam was sampled after the MOT, and focused onto a photodiode to provide spectroscopic confirmation of the EIT resonance. During all EIT experiments, the MOT repump laser was tuned to the \( F = 2 \rightarrow F' = 2 \) hyperfine transition to avoid additional coupling between states which would complicate the atomic coherence.

5. Results

5.1. Homogeneous object

A typical experimental image and reconstruction of column density is shown in Fig. 4. The raw diffraction pattern exhibits considerable noise and evidence of etalon fringes due to various optical surfaces in the beam line. To reduce the noise, a two dimensional Gaussian fit was performed on the unperturbed intensity \( I_0(r_2, z_2) \), recorded when the trap was empty. The noise, \( N(r_2) \), was defined as the difference between the fitted Gaussian, \( G_0(r_2) \) and the unperturbed intensity: \( N(r_2) = I_0(r_2, z_2) - G_0(r_2) \). Before solving the TIE for the phase of the waves, the noise was subtracted from the experimental images, resulting in a significant reduction of noise in the processed diffraction pattern:

\[
I'_0(r_2, z_2) = I_0(r_2, z_2) - N(r_2) \quad (13)
\]

\[
I'(r_2, z_2) = I(r_2, z_2) - N(r_2) \quad (14)
\]

Images demonstrated a good qualitative signal-to-noise ratio (SNR) using probe detunings between 0 and 3 \( \Gamma \) and defocus distances out to \( Z = 40 \text{ mm} \). At large defocus distances and
Fig. 4. Experimental results for Single-plane Curved-beam Phase Imaging. (a) Raw experimental diffraction pattern; (b) after noise subtraction; (c) reconstructed column density cropped to region inside intensity threshold, retrieved from diffraction pattern in (b), and (d) plot of column density through dashed line in (c). The blue line in (d) indicates the column density retrieved from a conventional in-focus, on-resonance absorption image taken on the same day. The diffraction pattern was recorded at a defocus distance of \( Z = 20 \) mm, a probe beam detuning of \( \Delta = 1.5 \Gamma \) and imaging probe pulse duration of 13 \( \mu \)s. A correction factor of \( \eta = 1.3 \) was used to obtain the density map (see Sec. 3). The circular discontinuity in the column density arises from the threshold function (Eq. (9)).

probe detunings, a high peak column density was required to produce a good SNR, however the recovered images consistently demonstrated good agreement with alternative quantitative imaging techniques, such as absorption [9] and diffraction contrast imaging [12]. For direct comparison of SCPI with conventional techniques, the quantitative density profile retrieved from an on-resonance absorption image taken on the same day is also shown in Fig. 4(d), showing excellent agreement with the SCPI result. The range of defocus distances investigated was only limited by optical access to the vacuum system and the size of the CCD. Further iterative refinement of the images using coherent diffractive imaging algorithms (e.g. Gerchberg-Saxton and error-reduction algorithms [36]) has the potential to provide enhancement of fine detail and structure within the cloud.

5.2. Inhomogeneous Imaging

In Fig. 4(b), etalon fringes reduce the signal-to-noise ratio (SNR) in the diffraction pattern, even after noise subtraction, due to shot-to-shot variation in probe laser frequency. To further reduce artefacts at higher spacial frequencies, including etalon fringes, the diffraction data for EIT imaging was spatially filtered by convolution with a Gaussian of variable full width at half maximum (FWHM), at the cost of slightly decreased spatial resolution.

Figure 5 shows experimentally acquired diffraction data and retrieved atomic refractive index of the atom cloud with spatial modulation due to the EIT pump beam, calculated using Eq. (5). The column density in Eq. (5) was measured with the pump beam blocked, and retrieved
Fig. 5. Imaging of an inhomogeneous atom cloud and extraction of the complex refractive index. Orange arrows indicate the propagation direction of the EIT pump laser beam. (a) Inhomogeneous diffraction data. The intensity of the unperturbed Gaussian has been subtracted for clarity. (b) Diffraction data from a homogeneous (pump beam off) cloud, unperturbed beam subtracted. (c) Column density reconstructed from (b). (d) Retrieved absorption coefficient, $\beta$, calculated using Eq. (5). (e) Retrieved phase coefficient, $\alpha$. (f) Experimental line profile (black) through dashed line in (d), with theoretical absorption curve shown in red. (g) Phase line profile, with theoretical curve, as in (f). Images were taken at $Z = 20$ mm defocus distance, $\Delta = 2\Gamma$ from the $F = 3 \rightarrow F' = 3$ transition. Theoretical lines were calculated using the exact version of Eq. (12), with parameters ($Q_1$, $\Omega_2$, $\Delta_1$, $\Delta_2$ and slit width) fitted to the data, within their respective experimental error margins. The data was convolved with a Gaussian of FWHM 186 $\mu$m prior to reconstruction.

Numerically using Eq. (4). The refractive index maps and line profiles have been smoothed using a five-pixel box-car average to reduce the effects of noise. Although this reduces the resolution of the result, our interest is in the variation of refractive index on the much larger scales of the atom cloud and laser beam profile.

The EIT pump beam is seen clearly as an area of low absorption and phase shift in a near-vertical region of the atom cloud. The extracted imaginary part of the refractive index shows good agreement with the theory. In regions of high atomic density, the discrepancy between measured and theoretical imaginary refractive index is less than 15%. In the outer regions of...
the cloud, where the density fell below $10^{11}$ atoms/m$^2$, Tikhonov regularization increases the maximum discrepancy to 40%. The mismatch between the retrieved and theoretical real part of the refractive index on the right-hand side of Fig. 5(g) is due to the hard edges of the EIT pump beam, caused by the slit through which it passed. Although the edges provide a more distinct and visible change in refractive index, they result in areas of rapidly varying phase which are not well accommodated by the approximations of this technique. For the Gaussian beams typically used in atomic coherence measurements, this artifact would be significantly reduced, if present at all.

Using the edges of the pump beam in Fig. 5(d) as a reference, we estimate the resolution of the refractive index retrieval as $\sim 40\mu$m. However, the diffraction data used to reconstruct that image was spatially filtered, and the reconstruction subsequently averaged over a five pixel radius, resulting in a substantial reduction in final resolution. In general, the technique does not impose any additional constraints on resolution, which is ultimately limited only by the probe wavelength and detector size. Furthermore, although Eq. (7) has been shown to work well for Gaussian objects, it can be adapted to suit more complex structures, relaxing the requirement for a smoothly varying object [1].

6. Conclusion

Single-plane Curved beam Phase Imaging (SCPI) was demonstrated, providing quantitative images of a cold atom cloud over a range of probe laser detunings and defocus distances. Images showed a good signal-to-noise ratio and quantitative agreement with conventional methods, as seen in Fig. 4(d). Unlike other non-iterative diffractive methods, SCPI can retrieve a quantitative image of the column density of an inhomogeneous object, given knowledge of the spatial variation in refractive index. By extracting the complex exit surface wave, the complex refractive index can be separated from the atomic column density. Either the refractive index or the column density can be recovered, provided that the other is known. Combined with the reduced sensitivity to experimental parameters outlined in section 3, SCPI can be a useful tool for a wide range of cold atom applications, including ultracold plasma diagnostics and studies of atomic coherence phenomena in ultracold gases. Additionally, the method could provide an excellent starting point for further iterative refinement of the images [36].

Simulation of imaging of cold atoms using typical experimental parameters yielded quantitative reconstructions of column density to within 10%. Using the results from the simulations, a detuning-dependent correction factor was calculated, which partially compensated for the errors introduced by the approximations used in the SCPI technique. Simulations also indicated an optimum region of the available experimental parameter space for imaging. The technique was implemented experimentally and the resulting images were of high quality compared to in-focus, on-resonance absorption imaging of homogeneous objects.

Although conventional absorption imaging remains one of the simplest methods of quantitative imaging, its sensitivity to the frequency of the probe laser beam and any defocus result in large uncertainties when used as a quantitative technique. Using SCPI, this sensitivity is significantly reduced by deliberately detuning the probe and measuring a diffraction pattern. Although other imaging techniques have demonstrated such reduced sensitivity [10], they are unable to separate the refractive index from the column density, limiting their use to homogeneous objects. SCPI retains the advantages of these techniques, while enabling imaging of atomic coherence phenomena in inhomogeneous cold atom clouds. Experimental reconstructions of an atom cloud with spatially modulated refractive index created using a cascade EIT scheme showed good agreement with theoretical predictions. Agreement to within 15% of theoretical predictions was achieved in areas of high atomic density where Tikhonov regularization did not affect the results. The resolution of spatially filtered and smoothed experimental images
of inhomogeneous clouds was estimated as $\sim 40 \mu m$, although the resolution of the technique is ultimately limited by wavelength and detector size.

This technique is particularly suited to imaging of atomic coherence effects in cold atom clouds, due to their typically Gaussian density profiles. Many experiments in atomic physics also utilize Gaussian laser beams. Accordingly, any induced inhomogeneities in the refractive index of the cloud will be smoothly varying, minimizing artefacts in quantitative reconstructions, such as those induced by the hard edges of the EIT pump beam in Fig. 5(g). Accordingly, SCPI offers the potential to yield greater information from atomic coherence experiments in cold atoms than commonly used probe beam techniques, while maintaining experimental simplicity.

We have shown an adaptation of the reconstruction process and experimental implementation of SCPI for cold atom imaging. The technique was used to image a complex object involving atomic coherence effects. Further work will involve application to electron bunch shaping in an ultracold plasma, and possible iterative refinement to increase fine detail. The simplicity of the technique makes it a useful tool not only for cold atom applications, but also for x-ray and phase imaging applications in general.