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Two-level network design with intermediate facilities: An application to electrical distribution systems ☆

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ABSTRACT

We consider the two-level network design problem with intermediate facilities. This problem consists of designing a minimum cost network respecting some requirements, usually described in terms of the network topology or in terms of a desired flow of commodities between source and destination vertices. Each selected link must receive one of two types of edge facilities and the connection of different edge facilities requires a costly and capacitated vertex facility. We propose a hybrid decomposition approach which heuristically obtains tentative solutions for the vertex facilities number and location and use these solutions to limit the computational burden of a branch-and-cut algorithm. We test our method on instances of the power system secondary distribution network design problem. The results show that the method is efficient both in terms of solution quality and computational times.

1. Introduction

Network design problems concern the selection of edges and vertices in a graph in order to satisfy, at minimum cost, some requirements, usually expressed in terms of the connectivity of the obtained network or its ability to allow a certain flow between source and demand vertices.

Several variants of network design problems have been proposed in the literature. They differ in terms of the number of commodities that must be transported, the existence of vertex or edge capacities, the presence of fixed or variable costs and the restrictions on the desired graph topology, to cite a few characteristics.

We consider that a single commodity must be transported from a root vertex to several demand vertices, via a radial network. There are two types of edge facilities which allow for the flow of the commodity, named primary and secondary. Primary edge facilities have higher fixed costs but smaller variable costs, when compared to secondary edge facilities. We call primary (secondary) flow, the flow of the commodity occurring in a primary (secondary) edge facility. The following additional restrictions must be respected:

(a) The root vertex must be connected to the network only through primary edge facilities.
(b) Demand vertices must be connected to the network only through secondary edge facilities.
(c) Primary flow can be converted in secondary flow only in vertices containing a costly and capacitated vertex facility. Moreover, some vertices might not be able to receive such facilities.

We name this problem the *two-level network design problem with intermediate facilities* (TLNDP-IF). The TLNDP-IF describes well an application in the context of electrical distribution networks. This problem will be detailed in the next section and is the main motivation for this work. Variants of the TLNDP-IF in which one or more of constraints (a)–(c) are slightly modified appear in many other network-engineering settings including, e.g., the design of telecommunication networks [1–6], the design of computer networks [7–9] and the design of reverse logistics networks [10,11].

In the design of telecommunication networks, the goal is to deliver broadband network services such as high-speed internet access, telephony or cable TV. The two-level structure appears due to the availability of different technologies such as optical fibers and coaxial links and the fact that intermediate facilities must be used to connect both technologies. In general, the demand vertices are served with coaxial links which originate at intermediate facilities. The intermediate facilities are, in turn, connected to a central root vertex by means of optical fibers. The used intermediate facilities can be of many types depending on the required capacities and functions. The design problem objective is to minimize the total cost of edge and vertex facilities

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that are used. As this problem is proved to be NP-complete, the solution techniques usually rely upon heuristics that hierarchically decompose the problem into smaller subproblems [5]. Lagrangian relaxation [12,13] and branch-and-cut [14] approaches have also been used.

In the design of computer networks, one is concerned with the definition of the topology of the network and with the traffic routing. The information must flow from central vertices to demand vertices through intermediate transmission facilities. The crucial decision is often where to install these intermediate facilities and how to connect them both to the central vertex(ices) (primary network) and to the demand vertices (secondary network). As before, a commodity or service has to be delivered from one or many source vertices to demand vertices through a two-level network, which has to be designed. In the primary part of the network, economies of scale induce the installation of large capacity edges—called the backbone or trunk network—while the delivery of the commodity to the demand vertices is made through less costly secondary edges, after a conversion is made in intermediate vertices. In these vertices a costly conversion capacitated facility must be installed. The quantity and location of these conversion vertices are also decisions that must be made during the network design.

As mentioned before, most of the solution techniques for these problems are heuristic approaches since the majority of the applications define hard combinatorial optimization problems. Very frequently the problem hardness forces a suboptimal hierarchical decomposition scheme, which means that the problem is hierarchically split into smaller interdependent subproblems. Details on hierarchical network design problems modeling and solution techniques can be found in [15–17].

In this work, we first propose a formal mathematical model for the TLNDP-IF. The model is used within a commercial MIP solver to obtain optimal solutions for small benchmark instances. In order to deal with real-world instances, the model is integrated into an efficient hybrid solution approach. We use a problem decomposition method which allows the obtention of good approximation for the locations of the conversion facilities by means of a Lagrangian surrogate technique. The heuristic solution is then used to restrict the number of candidate locations for the installation of intermediate facilities, in the flavor of what has been done in the context of vehicle-routing problems [18]. The restricted model is finally solved to optimality via a branch-and-cut method.

In the next section we present the application that has motivated this work: the planning of power system secondary distribution networks. Then, in Section 3, a mixed-integer linear formulation is proposed for the problem. In Section 4, we describe the developed hybrid decomposition heuristics, which are tested in Section 5. The paper ends with some conclusions in Section 6.

2. Application of two-level networks to the planning of power system distribution networks

We are particularly interested in an application appearing in the context of electrical distribution networks. The TLNDP-IF models well the situation faced when designing the so-called power system secondary distribution networks. In this section we describe the problem and present a brief literature review.

2.1. Problem definition

Distribution networks are the part of the power systems that connect the generation and transmission subnetworks to the final consumers. The voltage level is a criterion usually used to subdivide the distribution system network. In an upper level there is the primary distribution system and in a lower level, the secondary distribution system. These levels operate at different voltages, 13.8 KV and 220 V, for example, and are connected via street transformers. Fig. 1 shows the two levels schematically.

The secondary network design problem can be viewed in the enlarged new demand area in Fig. 1. Basically, the problem of planning such a system consists in defining (a) the transformers locations and nominal capacities, (b) the primary network links connecting the substations to the transformers and (c) the secondary network links connecting the transformers to the demand points.

The problem can be defined as a TLNDP-IF, where both levels are connected by the transformers. Therefore, a suitable location for the transformers plays a key role in the problem: they define the demand vertices for the upper level (primary network) and the source vertices for the lower level (secondary network). The three special constraints (a)–(c) presented in Section 1 are clearly required here, since:

(a) The substation delivers energy at medium voltages and, therefore, must be connected to the network through primary edge facilities.
(b) The consumers must receive the energy at low voltages (typically around 127 or 220 V) and, therefore, must be connected to the network via secondary edge facilities.
(c) The voltage conversion requires an electrical transformer, which has a capacity and a significative cost. These transformers can be installed in some of the electric posts.

Several notes are in order. (1) In the context of secondary network planning, vertices that do not allow for the installation of

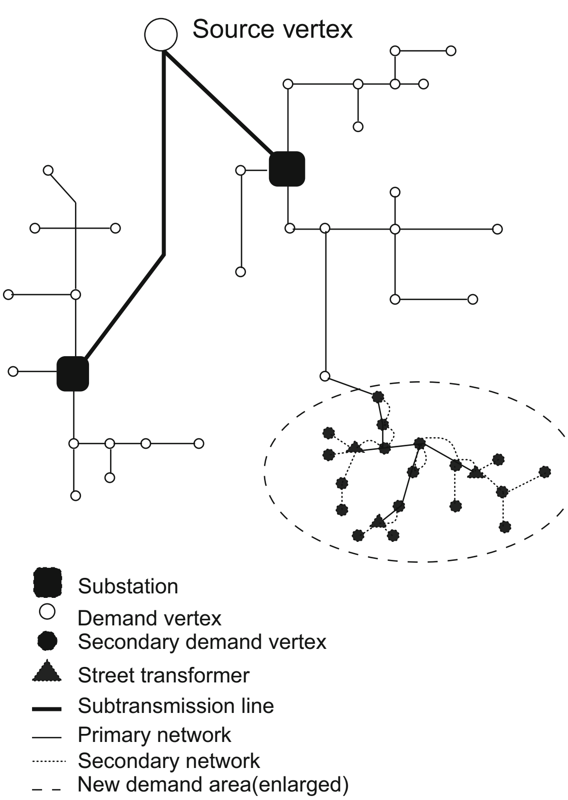

Fig. 1. Distribution network.

transformers are called flying-tap vertices. These vertices correspond to aerial connections and not to real electric posts and have no associated demand. (2) The size of the primary network that is dealt with in the secondary network design problem is relatively small when compared to the total size of the primary network. For this reason and also for the fact that the primary network operates at much higher voltages, the variable costs can be safely neglected in this network, in the context of this problem. (3) The variable costs of the secondary network are nonlinear but convex. These are usually the only costs considered in the secondary network since, for physical stability and operational reasons, all the secondary edges are installed (although many of them are kept disconnected from the network in order to ensure its radial structure). Notes 2 and 3 above indicate that, in the context dealt with in this article, only fixed costs are considered in the primary network and only variable costs are considered in the secondary network. This respects the general TLNDP-IF concept that says that primary networks have fixed costs that are higher and variable costs that are smaller, when compared to secondary networks.

Fig. 2 shows the sketch of a new demand area. The forecasted demand of each future consumer is known, as well as the electrical post to which each new consumer will be connected. Fig. 3 shows the graph that corresponds to the demand area under study. In the figure, note that the electrical posts that are already served by the primary network (in the case of Fig. 3, only the uppermost electrical post) will constitute the set of source vertices.

In Fig. 3 one can see a hypothetical demand associated with each post assuming that each consumer has a demand of 2 KVA. It is also shown all their possible interconnections. The optimal solution for this problem consists in the minimum cost network that supplies the forecasted load. Total cost includes the equipment costs (e.g., transformer and cable costs) and the nonlinear operation costs due to the electrical losses in the secondary network.

In order to compare solution alternatives comprising equipments with different lifetimes (transformers, cables) and taking into consideration the cost of losses, it is convenient to adopt the equivalent annual worth as a comparison measure. Table 1 presents the transformer costs for different transformer sizes as used by a Brazilian distribution utility [19].

For the transformers, it is interesting to note that the ratio cost/capacity experiences a huge drop when one goes from small nominal capacity transformers to greater ones. Therefore, considering only the transformers cost, it would be interesting to have a few large transformers in the network. However, a small number of transformers presupposes long secondary feeders (with higher losses cost). The optimal solution is, therefore, a trade-off dictated by the number of transformers and the length of the secondary network, as expressed in Fig. 4.

One possible solution for the secondary network design problem is shown in Figs. 5 and 6. Fig. 5 shows the new

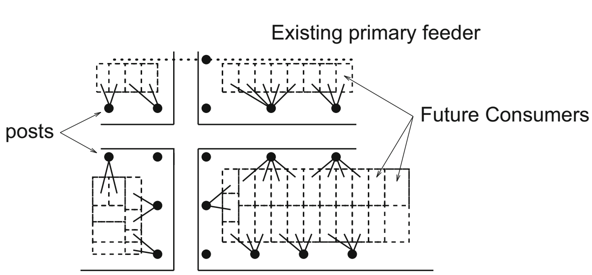

Fig. 2. New demand area.

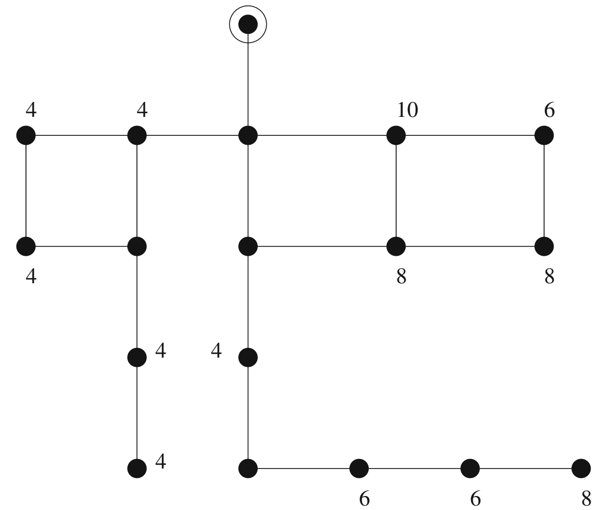

Fig. 3. Corresponding graph.

Table 1
Transformers costs according to their nominal capacity.

Capacity (kVA)	Cost (US$)	Cost/capacity (US$/kVA)
15	178.8	11.92
30	240.6	8.02
45	276.7	6.15
75	348.9	4.65
112.5	468.3	4.16

primary network feeders, connecting the existing primary network (represented by the uppermost vertex) to the transformers while Fig. 6 illustrates the secondary network, i.e., the low voltage feeders connecting the transformers to the demand points.

2.2. Literature review

A few authors have dealt with this problem. Aoki et al. [20] obtain a solution by limiting the candidate vertices for receiving conversion facilities. This oversimplification allows the authors to obtain feasible solutions by performing a local search on the edges. Carneiro et al. [19], in turn, divide it into the problem of taking three interrelated decisions, modeled as interdependent subproblems: (a) the location of the conversion facilities, (b) the routing, using primary-edge facilities, between the root vertex and the conversion facilities and (c) the routing, using secondary-edge facilities, between the conversion facilities and the consumers. The subproblems are solved by means of a *k-median* problem, a minimum spanning tree problem and a shortest-path problem, respectively. This division proved to be an intelligent choice allowing the authors to obtain good results in small computation times. The results obtained by Carneiro et al. [19] are used as a benchmark.

Similar problems have been solved by other authors. Diaz-Dorado et al. [21] have presented a dynamic programming approach that can only be used to solve situations with a small number of demand vertices while Diaz-Dorado et al. [22] developed an evolutionary algorithm, which deals with a population of forests. Each tree in the forest corresponds to a subnetwork

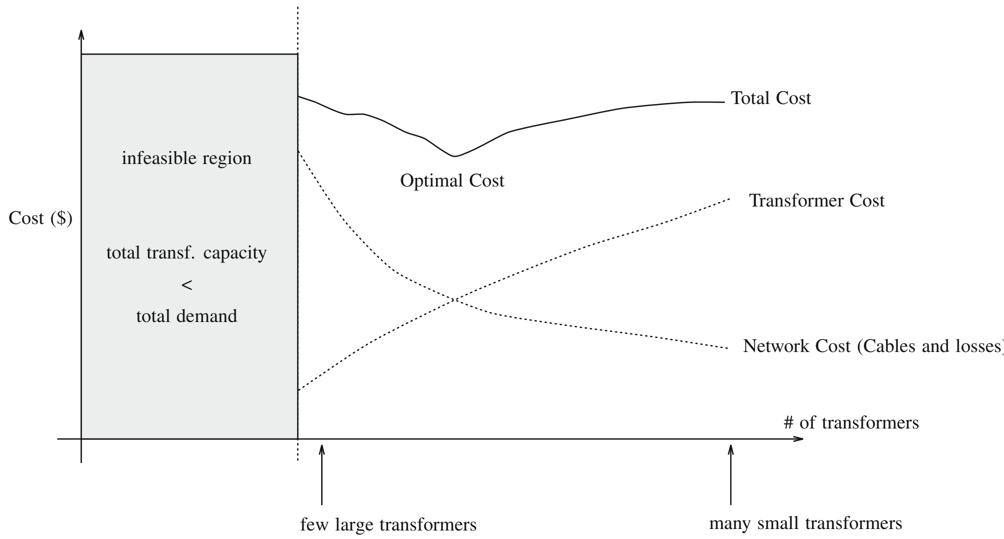

Fig. 4. Cost trade-offs.

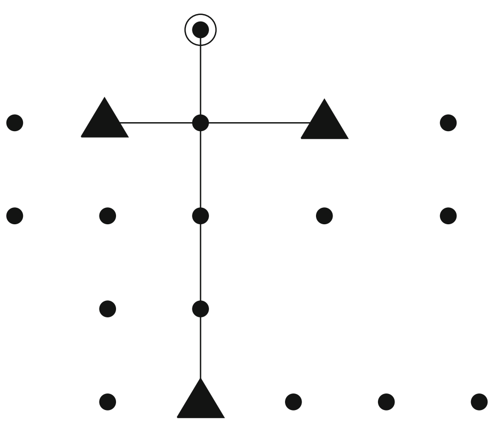

Fig. 5. Transformers location and primary network.

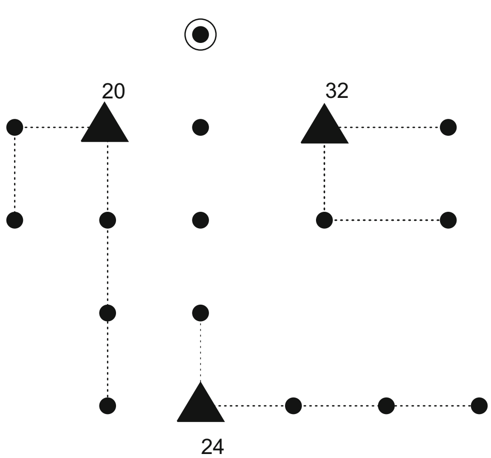

Fig. 6. Transformers associated demand and secondary network.

containing a single conversion facility and edges with secondary facilities. The evaluation of an individual is obtained by connecting the individual trees with primary edge facilities via a minimum spanning tree. Note that this is somehow similar to what has been done by Carneiro et al. [19], only that the authors use a different (and integrated) methodology for solving subproblems (a) and (c). Finally, Cossi et al. [23] propose an evolutionary algorithm for a similar planning problem which includes, however, some additional electrical constraints.

3. Mathematical formulation

The network design problem can be formulated as a mixed-integer nonlinear optimization program with a large number of integer variables. In order to do so, let us define the problem on a graph $G=(N\cup o, A)$, where N is the set of original vertices, o is an artificial vertex, and A is the set of edges. The subset $F\subset N$ defines the flying-tap vertices (which are, as defined before, vertices that have no associated demand and which cannot hold a conversion facility). Expressions (1)–(13) present a mathematical formulation for the power system secondary network design problem. In this formulation, we approximate the nonlinear costs associated with the electrical losses in the secondary edge facilities by a piecewise linear function.

$$\text{Min} \quad \sum_{(i,j)\in A, i<j} f^1_{ij} y^1_{ij} + \sum_{i\in N}\sum_{e=1}^{N_t} f^e z^e_i + \sum_{(i,j)\in A}\sum_{p=1}^{N_p} c^2_{ijp} x^2_{ijp} \tag{1}$$

S.t.

Flow conservation and transformer capacity

$$\sum_{j\in\delta^-(i)}\left(x^1_{ji}+\sum_{p=1}^{N_p} x^2_{jip}\right)-\sum_{j\in\delta^+(i)}\left(x^1_{ij}+\sum_{p=1}^{N_p} x^2_{ijp}\right) = \begin{cases} -\sum_{h\in N} d_h, & i=0 \\ d_i, & i\in N \end{cases} \quad \forall i\in N\cup 0, \tag{2}$$

$$0\le \sum_{j\in\delta^-(i)} x^1_{ji} - \sum_{j\in\delta^+(i)} x^1_{ij} \le \sum_{e=1}^{N_t} s^e z^e_i \quad \forall i\in N, \tag{3}$$

$$-d_i \le \sum_{j\in\delta^+(i)}\sum_{p=1}^{N_p} x^2_{ijp} - \sum_{j\in\delta^-(i)}\sum_{p=1}^{N_p} x^2_{jip} \le \sum_{e=1}^{N_t} s^e z^e_i - d_i \quad \forall i\in N, \tag{4}$$

$$\sum_{e=1}^{N_t} z^e_i \le 1 \quad \forall i\in N, \tag{5}$$

$$z^e_i = 0 \quad \forall i\in F,\ e=1\ldots N_t. \tag{6}$$

Link capacities

$$x_{ij}^1 \leq M^1 y_{ij}^1 \quad \forall (i,j) \in A, \ \ i<j, \tag{7}$$

$$x_{ji}^1 \leq M^1 y_{ij}^1 \quad \forall (i,j) \in A, \ \ i<j, \tag{8}$$

$$x_{ijp}^2 \leq M_p^2 \quad \forall (i,j) \in A, \ \ p=1 \ldots N_p. \tag{9}$$

Variable bounds

$$x_{ij}^1 \geq 0 \quad \forall (i,j) \in A, \tag{10}$$

$$x_{ijp}^2 \geq 0 \quad \forall (i,j) \in A, \quad p=1 \ldots N_p, \tag{11}$$

$$y_{ij}^1 \in \{0,1\} \quad \forall (i,j) \in A, \ \ i<j, \tag{12}$$

$$z_i^e \in \{0,1\} \quad \forall i \in T, \ \ e=1 \ldots N_t, \tag{13}$$

where x_{ij}^1 is the flow in edge (i,j) at level 1 (primary flow), x_{ijp}^2 the flow in the piecewise linear approximation segment p of edge (i,j) at level 2 (secondary network), y_{ij}^1 the binary variable associated with primary edge construction; y_{ij}^1 = 1 if primary edge (i,j) is built and 0 otherwise, z_i^e the binary variable associated with transformer installation; z_i^e=1 if a e-type transformer is installed at vertex i and 0 otherwise, c_{ijp}^2 the variable cost of edge (i,j) in the segment p (linearization for the secondary edge facilities). These costs increase as p increases. f_{ij}^1 is the installation cost of primary edge (i,j), f^e the installation cost of a e-type transformer, s^e the e-type transformer capacity, $\delta^+(i)$ the set $\{j|(i,j) \in A\}$, $\delta^-(i)$ the set $\{j|(j,i) \in A\}$, M^1 the primary edges capacity. M^1, is assumed to be large enough to supply the whole demand. N_t is the number of transformers types, M_p^2 the upper limit for the secondary flow in segment p of the linearization for the secondary edge facilities variable cost, N_p the number of linear segments in the linear approximation, d_i the demand at mode i.

The artificial vertex is connected with zero cost to all vertices representing electrical posts which are reached by the primary network (see Fig. 1). The production of this vertex is equal to the demand of all other vertices in the graph, i.e. ($d_0 = -\sum_{h \in N} d_h$), as stated in Eq. (2).

The objective function (1) minimizes the fixed installation costs associated with the primary edges and to the vertex conversion facilities, and the variable costs associated with the flow in the secondary edges. Constraints (2) guarantee the flow conservation. Constraints (3) and (4) assure that the transformer capacity is respected and that the flow transformation occurs always from level 1 (primary network) to level 2 (secondary network). Constraints (5) limit to one the number of transformers installed in a vertex and restrictions (6) forbid the installation of transformers in flying-tap vertices.

Constraints (7)–(8) deal with the edge capacities and ensure that there only exists primary flow in constructed edges. Since all secondary network is constructed, constraints (9) state that the flow on these edges can always exist, as long as the linearization limits are respected for each variable x_{ijp}^2. Constraints (10) and (11) forbid negative flows and (12) and (13) define variables y_{ij}^1 and z_i as integers.

As mentioned before, all secondary feeders must be installed, representing, therefore, an investment cost that cannot be optimized. This reflects in the formulation by the fact that there are no binary variables associated with secondary edge facilities. Nevertheless, the way the demand points and transformers are connected determines the flows in the secondary edges and, consequently, the secondary losses cost. Note that formulation (1)–(13) can be easily extended to the more general case presented in Section 1 with the inclusion of binary variables y_{ij}^2, representing the installation of a secondary edge.

4. Solution methodologies

In this section we describe three methodologies to solve the low voltage distribution planning problem. The first strategy (exact approach) is simply the resolution of formulation (1)–(13) via a branch-and-cut algorithm available in a commercial or open-source packages. As pointed out in previous works [19,21,22] the problem complexity induces a decomposition approach in which decisions are made in a hierarchical fashion. Following these ideas, the second methodology (decomposition approach) decomposes the problem into subproblems to obtain an initial solution and then uses a local search procedure to consider the interrelations between subproblems. Finally, the third resolution method (hybrid approach) tries to capture the best features of each one of the two first methodologies, by using the decomposition approach to reduce the size of the mixed-integer formulation, enabling its resolution with one of the mentioned optimization softwares.

4.1. Exact approach

The mixed-integer problem (1)–(13) can be solved with the aid of a mixed-integer algorithm. Due to the problem complexity, only small instances can be solved. The optimal solution values are used as benchmarks for evaluating other methods, as shown next in the computational comparisons.

4.2. Decomposition approach

The decomposition approach is based on the obtention of three interrelated sets of decisions: (D_1) location and sizing of transformers, (D_2) routing of primary feeders and (D_3) routing of secondary feeders. These three decisions are hierarchically connected as shown in Fig. 7.

As depicted in Fig. 7, the main decision is the location and sizing of the transformers. Indeed, once the transformers are fixed, the two auxiliary routings can be effected. The primary network routing sees the transformers as demand points while the secondary network routing has the transformers as the source vertices.

It is easy to integrate decisions D_1 and D_3 into the same solution approach with the aid of a *k-median* problem. Decision D_2, in turn, will be obtained with the aid of a *Steiner* tree problem. After a first initial solution is obtained, an improvement heuristic tries to change the position of the transformers in order to reduce the global cost. All these strategies are detailed in the following.

4.2.1. Obtaining D_1 and D_3

The first step of the method is to obtain D_1 and D_3. The *k-median* problem consists in finding the k best positions for the facilities (in our case, transformers) to serve the demand points. Each demand point is allocated to the closer median. The *k-median* cost is given by the sum of all "distances" from the demand vertices to the corresponding medians. As "distance", we do not use the distance from the vertex to the median but the electric momentum (distance to the median[1] × vertex demand). This is a very natural choice since the real cost in Eq. (1), $c_{ijp}^2 x_{ijp}^2$, considers not only the distance (implicit in coefficients c_{ijp}) but also the flow x_{ijp}. The *k-median* problem is formally presented

[1] Given by the shortest-path from the demand vertex to the median.

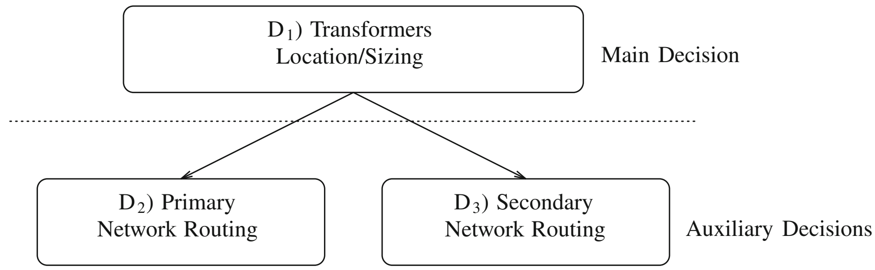

Fig. 7. Hierarchical solution approach.

in Eqs. (14)–(18).

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}t_{ij} \tag{14}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} t_{ij} = 1 \quad \forall i \in N, \tag{15}$$

$$\sum_{j=1}^{n} t_{jj} = k, \tag{16}$$

$$t_{ij} \le t_{jj} \quad \forall i,j \in N, \;\; i \neq j, \tag{17}$$

$$t_{ij} \in \{0,1\} \quad \forall i,j \in N, \tag{18}$$

where n is the number of vertices and $N = \{1, \ldots, n\}$; k the number of medians; d_{ij} the electric momentum between a demand vertex i and a median j; $t_{ij}(i \neq j)$ the binary variable associated with allocation; t_{ij} = 1 if the demand vertex i is served by median vertex j and 0 otherwise; t_{jj} the binary variable associated with the medians; t_{jj}=1 if the j th vertex is a median.

The transformer location/sizing decision is the core of the whole method. Therefore, a good solution to the *k-median* problem is crucial to produce good global solutions. On the other hand, (14)–(18) is still a NP-hard problem, which suggests the use of a heuristic approach. In this work we use a Lagrangian relaxation approach [24,25].

The rationale behind the Lagrangian relaxation is the observation that the *k-median* problem is a simple problem complicated by a small group of constraints. Indeed, if one dualizes constraints (15), the following problem results:

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{j=1}^{n}(d_{ij}+\lambda_i)t_{ij} - \sum_{i=1}^{n}\lambda_i \quad \text{S.t.} \quad (16),\ (17) \text{ and } (18). \tag{19}$$

The problem of minimizing (19) subject to (16)–(18) can be easily solved by inspection. The obtained solution (most likely infeasible) is a lower bound to the original problem. Moreover, we can apply a simple feasibility algorithm to obtain a feasible solution for the original problem.

The lower bound is improved by means of a subgradient procedure and when the difference between the lower and the upper bound attains a pre-specified error the method stops. The solution obtained by this algorithm indicates the k vertices that must receive a transformer. The size of a transformer is determined by the sum of the demands of the vertices allocated to the corresponding median in the *k-median* solution.

One must recall that the number of transformers k is also an optimization variable. To include this decision in the process while keeping the simplicity of the *k-median* approach, a batch is created and problem (14)–(18) is solved for a fixed k, starting at the minimum value necessary for a feasible solution (see Fig. 4) and repeating the procedure for k+1, $k+2, \ldots, k+n$, where n is a number large enough to include the optimal region of Fig. 4. Note that n can be obtained iteratively: the whole network configuration is obtained for k transformers, k+1, and so on. When the cost of the network increases for two successive interactions k+m and k+m+1, for example, we can stop the simulation, since the total cost curve in Fig. 4 is relatively well conditioned and resembles an unimodal function.

4.2.2. Obtaining D_2

Once the transformer locations are known, all entry data for the primary routing is available. Indeed, the decision of determining the primary network can then be modeled as the problem of connecting the transformer vertices to the primary source points (the electrical posts of the new area that already receive the primary network). As mentioned before, the variable costs associated with the primary network are neglected. Therefore, the goal associated with decision D_2 is to connect the source points to the transformers with the shortest possible network, which can be seen as the classical *Steiner* problem: given a graph where N is the set of vertices and P is a subset of these vertices, construct a minimum cost tree spanning vertices in P, using the vertices in N when convenient.

The *Steiner* tree problem has been solved with a simple procedure: first a complete graph with vertices P is considered. The distance between two vertices of this graph is given by the shortest path between the two vertices in the original graph. In this complete graph, a minimum spanning tree problem is solved, yielding an initial solution. This solution is then improved by inserting *Steiner* points, i.e., points in N that are not in P, but that can reduce the total length of the network. The procedure for searching and inserting *Steiner* points is rather straightforward: the algorithm scans all vertices in $N \backslash P$ with degree greater than 3 and analyzes the cost of the new tree including these vertices, one by one. The vertex leading to the best improvement (if any) is inserted in the tree and the remaining vertices are again scanned and their insertion cost computed. The procedure continues until no insertion improves the current network cost.

4.2.3. Improving the solution

Two major drawbacks of the approach just described are (1) the obtention of the important decisions D_1 and D_3 via an approximate *k-median* model and (2) the lack of integration on the obtention of the interdependent set of decisions D_1 and D_3 with the obtention of D_2.

The goal of this *improvement phase* is to cope with the second drawback. This is done by carrying out a local search considering the global cost of the problem, expressed by objective function (1). The local search uses a pairwise-interchange procedure to modify the position of the transformers and recalculates the global cost. If an improvement is verified the solution is updated, otherwise the changes are discarded, as shown in Fig. 8.

The way the transformers neighborhood is visited is guided by the Lagrangian relaxation solution to the *k-median* problem.

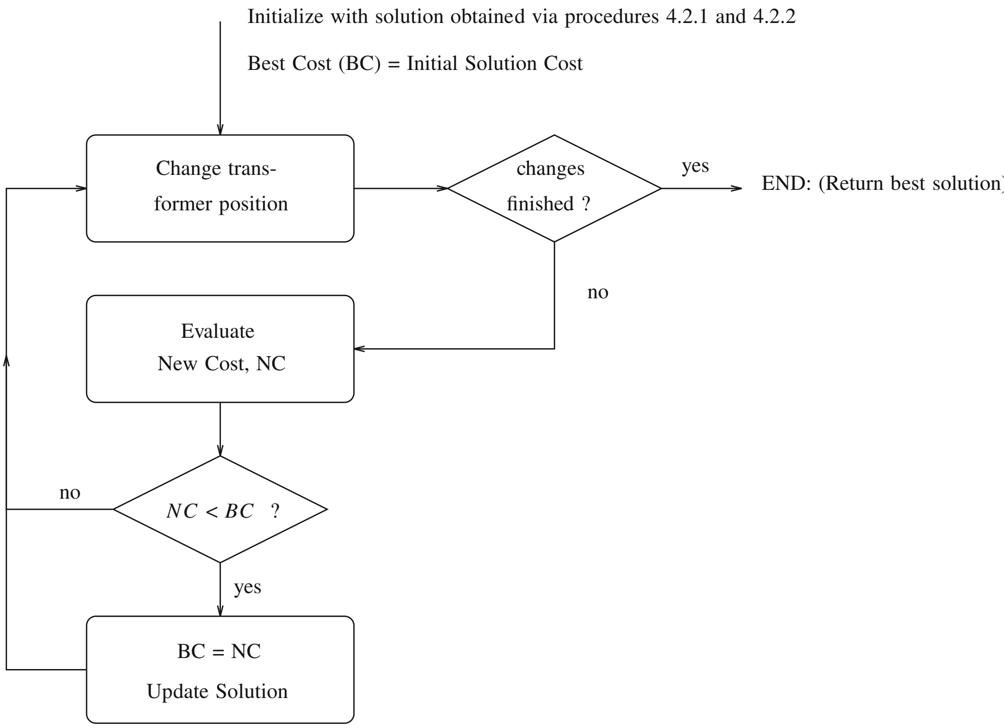

Fig. 8. Improvement phase.

Indeed, for each vertex i, the term ($\beta_i = d_{ij} + \lambda_i$) in the objective function (19) of the relaxed problem is somehow related to the quality of the vertex as a candidate for receiving a transformer. The pairwise-interchanges are made, therefore, giving priority to the vertices with the lower β_i values.

Once all possible changes have been tried since the last improvement, the algorithm is terminated.

4.3. Hybrid method

The exact approach presented in Section 4.1 gives the best possible solution for the problem formulated by expressions (1)–(13). However, this is done at an extremely high computational cost, prohibitive even for medium-sized instances. On the other hand, the decomposition approach presented in Section 4.2 is computationally efficient (as we shall see in Section 5 but obtains only near-optimal solutions.

The idea of the hybrid method is to combine the solution quality of the first method with the low computational times of the second. After a careful examination of the solutions obtained by the two methods, we came to the following conclusions:

- The transformer locations obtained by the *k-median* method were very close to the ones obtained by the optimal method.
- However, the *k-median* approach failed to obtain good secondary networks, because it does not explicitly consider the costs of electrical losses.
- Most of the time consumed by the optimal algorithm was due to the large number of integer variables related to the primary network selection and, in particular, to the transformers location/sizing.

These observations lead to the development of a hybrid algorithm that tries to use the good locations obtained by the decomposition algorithm to reduce the number of variables in the formulation to be used by an exact solution approach.

In a first stage, the *k-median* problem is solved and an approximative solution for the transformer locations is obtained. These locations and their closer neighbors become the only candidate locations when solving formulation (1)–(13), dramatically reducing the computational burden. This approach is similar to the notion of granularity introduced by Toth and Vigo [18] in the context of vehicle routing problems.

Fig. 9 exemplifies the idea: in Fig. 9(a) we see a sample solution obtained by the *k-median* algorithm. This solution is used to generate the candidate solutions that will be considered by the hybrid algorithm as shown in Fig. 9(b).

Three ways of defining the neighborhood were tested: (a) by considering all vertices directly connected to the solution vertices of the *k-median* problem (as in Fig. 9), (b) by considering all vertices located inside circles of ray r centered at the *k-median* solution and (c) by considering the k closest vertices to each solution-vertex in the *k-median* solution. Method (a) was found to be simpler and more efficient than the others and is used in this work.

The primary network is constructed as before, and therefore, the binary variables associated with the primary network are also eliminated in the 'exact part' of the hybrid method. It is worthy noting that eventual modifications on the primary network cost must be done during the solution of the hybrid formulation. This happens because the primary network is constructed based on the transformer locations obtained in the *k-median* problem, but these locations are not always kept during the branch-and-cut (one of its neighbors may be selected instead). Fig. 10 exemplifies all possible situations.

In Fig. 10, the locations selected by the *k-median* are represented by black triangles. The candidate solution vertices are represented by white triangles. The lines connecting the transformers compose the primary network. The three possible situations are:

(1) *Higher cost*: For instance, by choosing vertex 7 instead of 4, the primary network will have to be longer. The cost of the

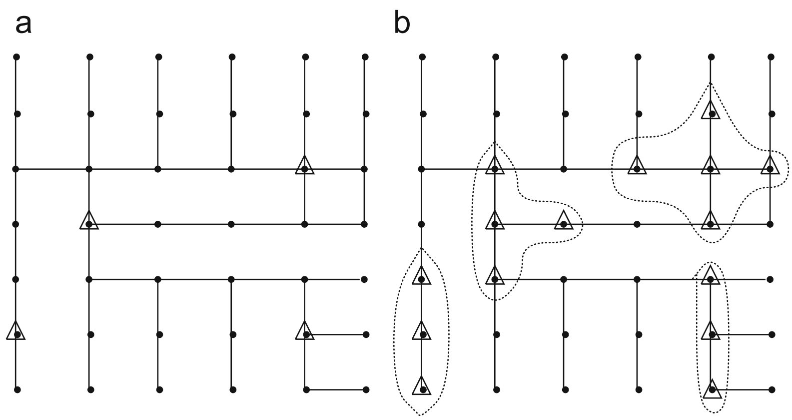

Fig. 9. Creation of a 'cloud' of candidate solutions surrounding the *k-median* locations. (a) Δ—solution of the *k-median problem*. (b) Δ—solution candidates in the hybrid algorithm.

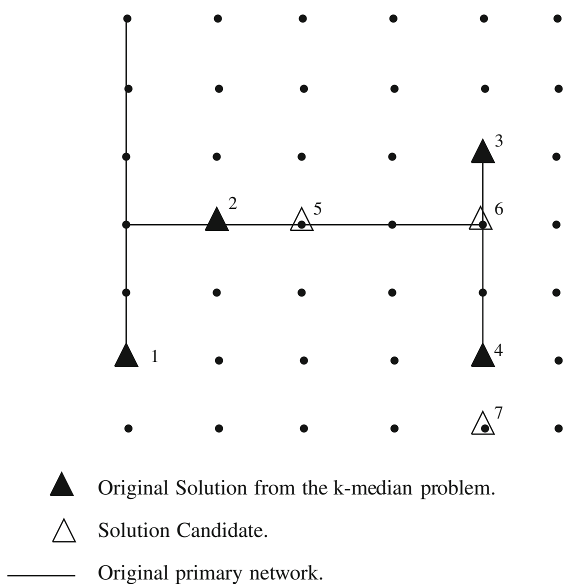

Fig. 10. Changes on the primary network according to the transformer locations selection.

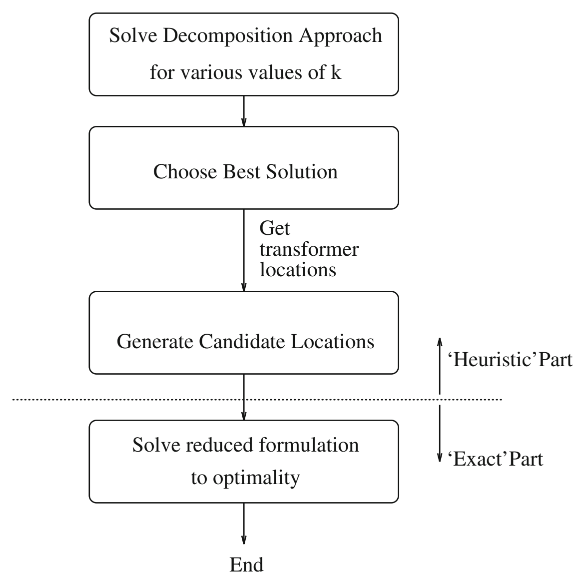

Fig. 11. Hybrid algorithm.

primary feeder connecting vertex 7 to vertex 4 must be added to the cost of selecting candidate location 4.

(2) *Lower cost*: By choosing, for instance, vertex 6 instead of vertex 3, the primary network has its length reduced. The cost of the primary feeder connecting vertices 3 and 6 must be reduced from the cost of selecting candidate location 6.

(3) *Same cost*: By choosing vertex 5 instead of 2: the primary network has its length and cost maintained.

Fig. 11 gives an overview of the hybrid algorithm. First, the decomposition approach is applied. Then, the resulting transformer locations are used to generate candidate solutions for the transformers locations and the reduced version of formulation (1)–(13) is solved to optimality.

4.3.1. Variants

An alternative to the algorithm presented in Fig. 11 would be to generate and solve a reduced formulation for each value of k, and not only for the value of k that yielded the best obtained solution. In our tests, the results obtained by this version of the algorithm proved to be slightly better but computationally much more expensive.

Other variants are also possible. The first one, consists in changing the first part of the algorithm. As depicted in Fig. 11, the first part of the algorithm ('heuristic part') obtains a solution for the transformers location/sizing problem. This can be done by any method, since the only information needed by the second part is the transformers tentative positions. We could use, for example, the solution obtained by other methods found in the literature, as the heuristic method proposed by Carneiro et al. [19].

Another reasonable variant of the proposed algorithm would be to further reduce the computational effort needed to solve the reduced formulation. One could think of reducing the size of the neighborhood or of completely eliminating this neighborhood, by considering as 'candidates' in the reduced formulation only the transformer locations obtained in the 'heuristic part'. The consequences are clearly predictable: we obtain a less accurate solution but in a shorter computational time.

In Section 5, in addition to the results obtained by the proposed methodologies we also effect computational experiments for three variants: (1) the hybrid algorithm using the method proposed by Carneiro et al. [19] as 'heuristic part', (2) a 'reduced hybrid method' where the solution of the heuristic part is fixed in the optimal resolution of the reduced formulation and (3) a combination of these two variations: the solution of Carneiro [19] is used in the reduced hybrid algorithm.

5. Computational tests

In this section we present the results to the computational tests that were carried out to evaluate the performance of the proposed heuristics. In Section 5.1 the test instances are presented, while in Section 5.2 the results themselves are presented. All simulations were carried out in a Sun workstation running Solaris.

5.1. Instance generation

We worked with three groups of instances. The first group is composed by small test instances randomly generated according to the methodology proposed by Aneja [26] in 1980. The used procedure is described below:

Procedure for generating a test instance with $|N|$ vertices and $|A|$ edges:

(1) Select $|N|$ vertices in the plane.
(2) Connect the vertices to form a tree.
(3) Add complementary edges to the tree, until the desired number of edges $|A|$ is attained.
(4) Allocate a demand to each vertex.
(5) Choose some vertices to receive the primary network.

The following criteria were used: (1) vertex coordinates are limited to integer values in the interval [0,100]; (2) the first tree is obtained in the following manner: vertex 1 is connected to vertex 2, which in turn is connected to vertex 3 and so on until vertex n is reached; (3) a vertex demand is randomly chosen in the interval [0,5]; (4) we assume that the primary network is always present on vertices 1 and 2.

With this methodology 20 instances (Tables 2 and 3) have been created and named *cbaxx* (where *xx* is the number of the instance). Using model (1)–(13), optimal solutions have been found. These values are then used as benchmarks to evaluate the proposed heuristics.

The second group is composed of eight geometrically regular small/medium-sized instances (Table 3). The idea is to represent a common situation where electrical posts are distributed almost uniformly in a grid. These instances are named *regxx* and were also solved to optimality.

Finally, the third group (*graxx*) is composed by two real cases, obtained from [19] and by one real network obtained from *Companhia Paulista de Força e Luz*, a Brazilian electricity distribution utility. These instances are presented in the end of Table 3.

Table 2
Test instances.

Instance	Group	$\lvert N\rvert$	$\lvert A\rvert$	Total dem. (kVA)
cba01	1	4	4	20.00
cba02	1	4	6	10.00
cba03	1	8	8	42.50
cba04	1	8	12	26.25
cba05	1	8	16	37.50
cba06	1	12	12	45.00
cba07	1	12	18	43.75
cba08	1	12	24	51.25
cba09	1	16	16	46.25
cba10	1	16	24	68.75
cba11	1	16	32	53.75
cba12	1	20	20	68.75
cba13	1	20	30	60.00
cba14	1	20	40	87.50
cba15	1	24	24	88.75

Table 3
Test instances.

Instance	Group	$\lvert N\rvert$	$\lvert A\rvert$	Total dem. (kVA)
cba16	1	24	36	92.50
cba17	1	24	48	86.25
cba18	1	30	30	125.00
cba19	1	30	45	122.50
cba20	1	30	60	113.75
reg04	2	4	4	15.00
reg09	2	9	12	45.00
reg12	2	12	17	55.00
reg16	2	16	24	80.00
reg20	2	20	31	100.00
reg25	2	25	40	100.00
reg30	2	30	45	150.00
reg35	2	35	58	175.00
grd1	3	100	100	51.56
grd2	3	173	192	750.00
grd3	3	143	153	605.00

Table 4
Results for group 1.

Instance	Opt.	Car	Dec	Hyb. variants			Hyb
				Var3	Var2	Var1	
cba01	251.74	0.00	0.00	0.00	0.00	0.00	0.00
cba02	261.53	10.21	7.73	10.21	7.73	0.00	0.00
cba03	526.16	2.84	1.27	2.84	0.00	0.00	0.00
cba04	389.96	2.15	0.13	2.15	0.00	0.00	0.00
cba05	511.79	1.66	6.30	1.66	6.01	0.00	0.00
cba06	758.43	4.11	0.80	4.11	0.00	0.00	0.00
cba07	846.50	3.72	3.72	3.63	3.63	0.00	0.00
cba08	655.15	0.11	0.11	0.00	0.00	0.00	0.00
cba09	1134.53	1.02	1.02	0.70	0.70	0.00	0.00
cba10	759.05	3.40	5.39	1.10	3.19	1.10	0.71
cba11	1134.59	1.05	0.97	0.17	0.09	0.09	0.09
cba12	1694.57	1.51	0.27	1.44	0.27	0.27	0.27
cba13	1240.70	4.97	3.41	4.74	3.29	3.29	3.29
cba14	1380.57	3.32	3.32	1.30	1.30	0.00	0.00
cba15	2151.42	0.56	0.38	0.56	0.32	0.07	0.07
cba16	1641.10	2.70	1.18	1.05	0.00	1.05	0.00
cba17	1396.60	5.62	2.26	3.42	0.40	1.47	0.06
cba18	2666.29	2.64	0.99	1.85	0.00	1.15	1.15
cba19	1889.40	3.15	1.75	1.15	0.15	1.15	0.15
cba20	2106.99	1.69	3.70	1.16	0.00	0.00	0.00
Mean		2.82	2.24	2.16	1.35	0.48	0.29

5.2. Results

The two developed algorithms have been applied to the instances presented in the last subsection and the results compared to the optimal results (when available) and also to the solutions obtained with the method by Carneiro et al. [19]. Moreover, the three variants described in Section 4.3.1 were also analyzed.

Tables 4–6 present the results for the three groups of instances. The columns in the tables represent the following:

- Opt. The optimal solution obtained with the commercial package *Cplex* (except for the two last rows of Table 6, for which no optimal solution could be found and we use the best value found by any of the other methods).
- Car. Results of [19].
- Dec. Results for the decomposition method of Section 4.2.
- Hyb. Results for the hybrid method of Section 4.3.

Table 5
Results for group 2.

Instance	Opt.	Car	Dec	Hyb.variants			Hyb
				Var3	Var2	Var1	
reg04	164.00	2.29	2.29	2.29	2.29	0.00	0.00
reg09	448.50	0.56	0.28	0.00	0.11	0.00	0.00
reg12	686.50	1.18	1.18	0.00	0.00	0.00	0.00
reg16	824.50	12.95	0.61	6.60	0.61	0.61	0.61
reg20	613.75	3.67	3.95	0.10	0.43	0.10	0.10
reg25	1026.29	1.83	1.83	0.00	0.00	0.00	0.00
reg30	935.25	7.85	6.76	1.30	0.24	0.50	0.24
reg35	1057.75	19.40	8.27	10.13	0.30	1.49	0.00
Mean		6.22	3.15	2.55	0.50	0.34	0.12

Table 6
Results for group 3.

Instance	Opt.	Car	Dec	Hyb. variants			Hyb
				Var3	Var2	Var1	
grd01	1301.98	4.27	4.27	2.08	2.08	0.00	0.00
grd02	7958.18	0.98	0.71	0.26	0.31	0.00	0.10
grd03	8029.24	2.90	1.44	1.25	0.73	0.84	0.00
Mean		2.72	2.14	1.20	1.04	0.28	0.03

Var1. Variant 1 of the hybrid approach which consists in obtaining the transformer positions with the method presented in [19].
Var2. Variant 2 of the hybrid approach which consists in further reducing the optimal formulation by fixing the transformers at the positions obtained by the heuristic method.
Var3. Variant 3 (mix of variants 1 and 2) of the hybrid approach which consists in further reducing the optimal formulation by fixing the transformers at the positions obtained by the heuristic method, in this case the heuristic presented in [19].

In the first column, the cost is presented in dollars while in the other ones we present the percentage deviation from the optimum.

Table 7 presents the computational times in seconds expended by each method.

Table 4 shows that, in the average, our decomposition method reduces the deviation in about 0.6% for instances of groups 1 and 3 and 3% for the instances of group 2.

The quality of the proposed solution approach can be also observed by noting that the hybrid approach (and its variants) obtains better results when its initial solution is used, in comparison to the cases where it is initialized with the solution obtained with the heuristic of Carneiro et al. [19]. Indeed, this can be observed if one compares columns Var2 (reduced hybrid approach starting with the proposed method) and Var3 (reduced hybrid approach starting with the method of Carneiro et al. [19]) or columns Var1 (hybrid approach starting with Carneiro et al. [19] heuristic) and Hyb (hybrid approach starting with the proposed decomposition approach).

The price to be paid for these better results is a larger computational time. However, the method is still very efficient and can solve real instances in about 1 min. Note that this increase

Table 7
Computation times (s).

Instance	Car	Dec	Hyb. variants			Hyb
			Var3	Var2	Var1	
cba01	0.03	0.12	0.03	0.13	0.08	0.13
cba02	0.04	0.07	0.05	0.08	0.11	0.08
cba03	0.04	0.14	0.05	0.15	0.07	0.17
cba04	0.04	0.14	0.05	0.15	0.06	0.18
cba05	0.04	0.13	0.06	0.14	0.09	0.16
cba06	0.04	0.14	0.05	0.15	0.07	0.18
cba07	0.08	0.17	0.10	0.18	0.13	0.21
cba08	0.05	0.19	0.07	0.20	0.12	0.24
cba09	0.07	0.27	0.10	0.31	0.32	0.53
cba10	0.07	0.23	0.09	0.25	0.13	0.28
cba11	0.08	0.22	0.10	0.24	0.15	0.30
cba12	0.06	0.17	0.11	0.20	0.40	0.55
cba13	0.08	0.23	0.09	0.26	0.16	0.31
cba14	0.07	0.29	0.12	0.36	0.37	0.59
cba15	0.08	0.27	0.16	0.34	1.32	1.54
cba16	0.09	0.31	0.15	0.37	0.53	0.63
cba17	0.09	0.33	0.16	0.42	0.51	0.78
cba18	0.12	0.39	0.20	0.47	4.67	6.89
cba19	0.10	0.32	0.17	0.38	0.42	0.70
cba20	0.11	0.47	0.22	0.55	0.44	0.95
reg04	0.03	0.05	0.04	0.06	0.06	0.08
reg09	0.06	0.20	0.07	0.21	0.11	0.25
reg12	0.09	0.26	0.10	0.28	0.13	0.32
reg16	0.11	0.39	0.16	0.41	0.29	0.61
reg20	0.12	0.44	0.18	0.51	0.52	0.82
reg25	0.13	0.51	0.21	0.59	0.52	0.90
reg30	0.15	0.47	0.25	0.55	1.12	1.68
reg35	0.17	0.38	0.54	0.52	6.26	1.94
grd01	0.39	1.05	0.93	1.59	0.79	1.80
grd02	2.61	75.37	4.75	77.51	352.35	416.83
grd03	1.27	54.00	2.44	55.54	507.38	760.48

in the computational time is present in all hybrid methods that start with the proposed decomposition approach.

In what concerns the proposed hybrid approach, the method is able to find 17 out of the 29 optima. Moreover, in 27 out of 31 instances, the hybrid approach was able to improve the solution obtained by the decomposition method.

Regarding group 3, the hybrid method found a proved optimum for the smallest real instance and very good solutions (in comparison to the other methods) for the other two, decreasing the deviation from the optimal solution in up to 4.27%, when compared to the results found in the literature. It is interesting to note that the results used as benchmarks [19] had already presented improvements of up to 13%, when compared to the heuristic procedures used at the studied electricity distribution utility (CPFL).

Concerning the computational times, we note that they are considerably increased by the resolution of the mixed integer formulation. However, one must highlight that even for the slowest method and the largest instance, the computational times do not exceed a few minutes, which seems reasonable in a strategic planning problem such as the one dealt with here.

6. Conclusions

We propose a hybrid decomposition approach for the planning of two-level networks in which the two levels must be interconnected through intermediate facilities. This problem arises in many network-engineering contexts. Our approach decomposes the problem in the location of the intermediate facilities, which is followed by routings in both network levels. Since the location of the intermediate facilities is a critical decision, we propose a hybrid exact–heuristic approach in which the heuristic solution is used to restrict the number of variables in the 'exact' part, obtaining a trade-off between solution quality and computational time. We test our algorithms in the planning of electricity distribution networks and the results show that the proposed methodologies are relevant, obtaining gains of up to 4.27% in real networks when compared to the results presented in previous works.

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