# Exploring the Utility of Crutch Force Sensors to Predict User Intent in Assistive Lower Limb Exoskeletons

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Abstract—The adoption of assistive lower limb exoskeletons in built environments is reliant on the further development of these devices to handle the varied conditions experienced in everyday life. The required development includes more varied and flexible gait patterns, but also appropriate user interfaces to enable fluid gait. This work explores the properties of an algorithm used to predict user intent based on sensors onboard a user-balanced robotic exoskeleton system. Specifically, classification algorithms built with different input data sets are compared — with varying detail of the interaction forces between the crutches and the ground, and the duration of the data sample used to make the prediction. Data were collected with one able-bodied participant using an exoskeleton, training three independent classifiers corresponding to different exoskeleton states. The results indicate the value of including information about the interaction forces between the crutches and the ground in improving prediction accuracy, with increasing prediction window also generally resulting in an increase in prediction accuracy. Whilst no categorical recommendation can be made with respect to either parameter, these results provide a baseline which can be used in conjunction deliberate consideration of the costs associated with implementation.

# I. INTRODUCTION

The development of lower limb exoskeletons to assist those with movement impairments to ambulate has seen great strides in the last two decades, with technological improvements allowing the development of numerous research and commercial devices. Whilst these devices have seen adoption within controlled clinical settings, use within assistive - every day - environments remains limited. Although there are numerous issues to overcome, it remains that exoskeletons are currently incapable of producing the variations in movements required for traversing everyday life [1], [2]. Achieving these movements is a multi-faceted problem: developers must derive exoskeleton movement patterns to ensure that the movements can physically be performed safely, but, simultaneously, user interfaces must be developed to account for the myriad of new options in a fluid, natural way. In this work, we explore the use of force sensors in crutches to assist the pursuit of this second goal.

Typically, assistive exoskeletons have a number of set movements (e.g. walking or sitting from a standing posture), with the user using buttons to select from the options presented on a screen. Scaling these interfaces to accommodate many more movement patterns is impractical due to the number of button presses required. Therefore, exploration towards other modes of identifying user intent is warranted.

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Important to the adoption of such interfaces is usability and practicality. For example, donning and doffing time has been identified as another factor limiting assistive exoskeleton adoption [1], and thus solutions utilising systems integrated into the exoskeleton system are preferred. This limits the use of biosignals such as electromyography (EMG) or electroencephalogram (EEG), as in [3], [4], which require careful and time-consuming placement of additional sensors on the user's body. Furthermore, techniques requiring that the user has some voluntary control of their legs, such as kinetic sensors measuring the interaction force between the user's leg and the exoskeleton [5], are also not suitable, given users of assistive devices do not necessarily have this control.

In user-balanced exoskeletons, crutches are used to maintain balance, but also to affect subtle changes in gait for example, turning or small variations in foot placement. Thus, it is hypothesised that data from these crutches may be useful in development of scalable user interfaces. Whilst such signals have been used in threshold-based algorithms [6], such strategies become difficult to employ when more movement patterns are added. This work seeks to specifically explore their use within data-driven classifiers, which may be more easily scalable. Whilst this prevents application to exoskeletons not requiring crutches (*e.g.* [7]), it is clear that user-balanced exoskeletons are closer to real-world adoption, and thus studies specific to these devices are justified.

As such, this preliminary work investigates the characteristics of a movement classifier which predicts the user's next intended movement. Specifically, this work sought to understand the value of including crutch/ground interaction forces in improving the prediction, comparing classifier performance when data from different sensors are included. This is achieved through a data collection experiment with a single able-bodied participant using a lower limb exoskeleton with limited movements, and manually-controlled movement transitions; and training and evaluation of classifiers using differing subsets of the collected data.

# II. BACKGROUND

# A. Intention Detection in Assistive Exoskeletons

User Interfaces in assistive exoskeletons have traditionally been quite simple, with limited options in their control modalities requiring limited user interfaces. A recent review [8] reported that approximately half of the exoskeletons in the world follow predefined trajectories, with the other half modifying these trajectories based on the interactions between the user and the device.

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Choosing between predefined movement trajectories has generally been achieved either through the use of buttons or other manual user interface, or by using discrete triggers (thresholds) based on crutch sensors, in-foot pressure sensors and centre of mass predictions. This has provided distinct rules to initiate a limited number of step types [6], where thresholds were used to trigger transitions. Similarly, the Indego exoskeleton (Parker Hannifin Corporation, USA) initiates different movements based on an estimate of the position of the user's centre of pressure (CoP). Such approaches become less viable with the introduction of more and more movements — as any thresholds become closer and closer together, which increases the likelihood of an automatic initiation of any undesired movement.

On the other hand, works in which trajectories are continuously adjusted during their execution have explored the use of additional signals - particularly biosensors, such as electromyography (EMG) or electroencephalogram (EEG). Such techniques often attempt to modify the movement pattern based on muscle activations during gait, such as in the Hybrid assisted limb (HAL) (Cyberdyne Inc., Tsukuba, Japan), and the works reviewed in [9]. However, such signals are difficult to integrate into practical assistive devices, due to the time required to place the sensors each time the device is to be used, and the large amount of human-to-human variability and sensitivity to sensor placement. Whilst some of these problems can be overcome with purely mechanical sensors — such as a using onboard sensors (motor torques) to change gait speed [10] — such approaches require some function of the lower limbs, reducing their viability for those who have lost voluntary limb control. A recent novel approach used a mechanism of using "poles" (or crutches) to control the position of the foot [11]. In that work, force sensors embedded in poles are used to virtually 'pull' along the foot. This clever approach allows for a high degree of continuous control over feet positions. However, it is also potentially cumbersome as the hands are now completely occupied to control the feet, which may have implications for maintaining balance in user-balanced exoskeleton devices.

The goal of the present project is to explore techniques for switching between movement trajectories, which can be applied to cases in which the predefined trajectories are more similar than those explored in the literature, for example, a short or long step. It is hypothesised that a small number of additional movement trajectories, combined with the user's ability to influence trajectories using their own dynamics [12], will significantly improve the variability that the human-exoskeleton system is capable of handling.

# B. Proposed Operational Approach

The proposed approach for controller the exoskeleton is as follows: based on the set of available actions, an algorithm using information from the user's movements is used to identify the action which the user is most likely attempting to execute. The selected movement is presented to the user for confirmation (for example, by pressing an easily-accessible crutch-mounted button), who then initiates the movement with certainty, see Fig. 1.



Fig. 1. Operational approach proposed in this work. In a given state, algorithm continuously makes predictions of the next desired user action from the set of possible actions, based on a sliding window of data. The "current" prediction is actioned upon confirmation by the user.

A data-driven approach is ideal for this algorithm, as it enables the classifier to capture the 'natural' actions of the user as that movement is attempted. A similar approach has been taken with lower limb prostheses, where information from similar sensors was used to predict the intended step type for the user [13]. It is noted that the use of data-driven algorithms is prevalent in the related field of prostheses, commonly referred to as locomotion mode recognition (LMR), with works demonstrating success of the approach using on-board sensors include inertial measurement units, force sensors, motor torques and camera-based data [14], [15].

## III. MATERIALS AND METHODS

In this section, the experimental platform on which the study was conducted is introduced; along with methods used to capture the input data; and the techniques used to create and evaluate the support vector machine-based classifiers.

#### A. Exoskeleton System

A modified ExoMotus X2 (Fourier Intelligence, China) was used for this work. This lower limb exoskeleton has four active degrees of freedom (hip and knee flexion/extension) and is driven by a custom application developed using CANOpen Robot Controller (CORC) [16].

In the application, the set of allowable movements was limited, and is represented in state diagram form in Fig. 2. For each stationary pose (state), a movement (transition) is used to move to another pose. The software was configured to allow standing from a seated position, forward and backwards steps, and for the user to bring their feet together. The three poses of interest in this work are the standing pose (3 possible movements), the left foot forward (LFF) pose (2 possible movements) and the right foot forward (RFF) pose (3 possible movements). This limited state machine was chosen as a test case for this study in order to understand feasibility of the approach, as the three different classifiers required different numbers and types of available movements. As a result, the state machine notably included asymmetric LFF and RFF states, and excluded some movements likely required for everyday use - such as stepping forward with the right foot when in the standing state.

The system was driven by a crutch-mounted user interface. A screen displayed the current state (pose) and the currently selected movement. When the exoskeleton was in a stationary pose, the user could scroll through the available movements using physical buttons, before initiating and executing the movement with a trigger under the index finger. Importantly, this trigger provided a 'ground truth' around the timing of the user's intention – it is at the point the user initiates the movement that the decision is made, and thus it is hypothesized that the information available immediately before the trigger is pressed can be used to predict the intention of the user at that moment in time.



Fig. 2. State machine representation of the possible movements within this work.

Two sets of sensors were available (see Fig. 3):

- *Exoskeleton Motors:* Each actuated joint on the exoskeleton (The hips and the knees) were driven by Copley ACK-055-06 Accelnet Micro Module Motor drives (Copley Controls, USA) powering Maxon EC60 Motors (Maxon Group, Switzerland), coupled with a belt and pulley and gearbox. The drives output motor position, velocity and torque information.
- *Crutch Forces:* Robotous RFT80-6A01-A sensors (Robotous Inc, Korea) were installed at the end of each crutch. These sensors output both force and torque information in three dimensions.



Fig. 3. Location of sensors used for data collection.

### B. Parameters of Interest

With the goal of the system being to predict the intention of the user, the present investigation aimed to investigate effect of two main parameters regarding the input data to the system: the use of sensors measuring the interaction force between the crutches and the ground, and the length of the prediction window. These parameters have practical implications for the implementation of a more complex system capable of classifying between a larger number of classes. Whilst it was expected that these parameters would have an effect on the performance of the classifier, the present investigation sought to explore the effects' magnitudes, to identify any clear cost/performance trade-offs.

1) Sensor Set: Exoskeleton actuators generally have some means of reporting their level of actuation. Whilst there is likely a non-trivial amount of transmission loss before the joint, it is hypothesised that this 'free' source of data may provide information about the pose of the user, and thus their intention. This provides the baseline data set.

In addition, we also sought to understand the value of including crutch-ground interaction kinetics. Data from the Force/Torque sensors at the ends of the crutches were also considered, along with subsets of this data, with the hypotheses that (1) this information may improve classification accuracy, and (2) simpler, less expensive sensors may provide similar results. Thus, the available data sets were:

- CM: Crutch Moments
- CF: Crutch Forces
- CFZ: Axial Crutch Force (z-axis only)
- MT: Motor torques
- And models were evaluated with the following sets of data:
  - [CM + CF + MT]
  - [CF + MT]
  - [CFZ + MT]
  - [MT]

where the [MT] set represents the case when no F/T sensors are included in the system, with other options considered including a single-axis load cell (CFZ), a tri-axis load cell (CF), and the full F/T sensor (CM + CF).

2) Prediction Window: As a second objective, the proposed approach required a "rolling" prediction, which updates continuously until an action is confirmed. This investigation also sought to explore whether increasing the length of the prediction window would significantly improve the results. This has implications for the delay before the first prediction is offered, as well as the complexity and performance requirements on the hardware running the classifier.

### C. Data Collection

One able-bodied individual with no known underlying neurological conditions was recruited for this study (Male, 22years old). After familiarization with the operation of the exoskeleton, the individual was asked to perform a series of movements between each of the states, over experimental sessions over two days. Movement direction was random, with each step within an arc of approximately 120 degrees (60 degrees either side) in front of the user. It is to note that the exoskeleton does not have degrees of freedom in these directions, and thus turning maneuvers were achieved using force from the user's upper limbs. Reflecting real-world use, the participant was aware of the plans for each subsequent movement prior to the completion of the previous.

Data from each of the sensors was logged at 100Hz, along with the exoskeleton state and trigger state, on a dedicated logging device running a custom logging application. At completion of the data collection, data were segmented into pre-movement periods, for each state-transition pair, using the data representing the trigger press as a marker. State information recorded from the exoskeleton software was used as indicators for the state and transition. This provided a full dataset, subsets of which were then used to train different classifiers to understand the important characteristics of such.

#### D. Classifier Development

To provide a mode of comparison, a consistent method of producing a classification classifier was constructed. At its core, this classifier used a Support Vector Machine (SVM) classifier to predict the movement intention from the available data. This work leveraged scikit-learn [17] for the construction of the classifier.

1) Data Scaling and Dimension Reduction:

a) Scaling: For this initial investigation, all components of the input data were considered of equivalent importance. Thus, StandardScaler was used to standardize the effects of any given input parameter.

b) Dimension reduction (PCA): In addition, a Principle Components Analysis (PCA) was used to reduce the dimensionality of the dataset. The goal was to reduce the complexity of the classifiers, particularly those with larger input datasets. PCA was applied to the entire sample (thus choosing more informative channels over others), reducing the dimensionality to the point which captured 80% of the variability. Whilst a relatively arbitrary choice, it provided a common standard across all prospective classifiers.

2) Support Vector Machine: Support vector machines (SVM) were chosen over a neural nets due to their improved performance for smaller training datasets – which was of interest given the possibility that a unique classifier may need to be created for each individual user. In cases where a non-binary classification was required, a one-vs-one strategy was employed. To accommodate the likely scenario that the datapoints could not be separated using a linear boundary, a kernel function was used. The exact kernel function used was left as a parameter to be optimized, using GridSearchCV, between the commonly selected kernel functions types of linear, radial basis, polynomial, and sigmoid.

## E. Evaluating Classifier Performance

Classifiers were compared on accuracy – i.e. the classifier's percentage of correct predictions against total predictions. However, to understand the level of generalisability of the classifier, the process was performed 12 times, with a randomized train/test (75%/25%) data split each time the

classifier was created and evaluated. This ensured that any particular evaluation of the classifier (or approach) was not biased by any particular test/train split.

## IV. RESULTS

In total, data for 717 movements were collected, with a distribution which can be seen in Table 1.

TABLE I Total Data Points Collected

State	Transition	Movements Recorded	Movements in State
Standing	Sitting Step Forward	26 105	215
	Step Back	84	210
Left Foot Forward	Step Forward Feet Together	147 121	268
Right Foot Forward	Step Forward Feet Together Step Back	80 52 102	234

## A. Stand Classifier



Fig. 4. Performance of the Stand Classifier. Error bars represent one standard deviation. Data points separated for clarity on error bars only — prediction window times were set at multiples of 100ms.

The Stand classifier produced an accurate response across all input datasets, with all classifiers across all prediction windows and all sensor sets producing a classification error of less than 1.5% on average.

#### B. Left Foot Forward Classifier

The Left Foot Forward (LFF) classifier showed large improvements in performance with each additional sensor data set added (4.5%, 7.1% and 4.7% for axial force, all force and moment interactions respectively). In addition, the performance of the classifier improved with a longer prediction window in almost all cases, from the 100ms to 1000ms range explored.



Fig. 5. Performance of the Left Foot Forward Classifier. Error bars represent one standard deviation. Data points separated for clarity on error bars only — prediction window times were set at multiples of 100ms.



Fig. 6. Performance of the Right Foot Forward Classifier. Error bars represent one standard deviation. Data points separated for clarity on error bars only — prediction window times were set at multiples of 100ms.

# C. Right Foot Forward Classifier

The Right Foot Forward (RFF) classifier saw a larger increase in performance when the bare minimum crutch axial forces were added (5.9% on average), with smaller improvements with additional crutch force and moment data. Increasing the prediction window in this classifier did not appear to significantly improve the performance.

# V. DISCUSSION

# A. Classifier-Specific Comments

The performance of the Stand classifier was very good across all conditions. This is unsurprising, given that the three movements are very different, and thus easily distinguishable. Interestingly, the same level of performance was observed with only the motor torque information, suggesting that the crutch information was not required. However, given that this classification problem can intuitively be solved with simple thresholds (as in [6]), the performance of this classifier does not significantly influence the implications for this work, with the exception of validating the pipeline. The performance of the LFF classifier increased almost monotonically with additional input data, as well as prediction window. However, the classification error is as low as 5% with all available data. This performance is promising with respect to being able to differentiate between smaller step sizes, due to the fact that the two options for this classifier are effectively a 'normal' step and a 'small' step.

The lower performance of the RFF classifier is surprising, especially when considered along side to the LFF classifier, as the additional movement considered in the RFF is in the opposite direction. This is explored further in Fig. 7, which presents the average confusion matrix for the [CM+CF+MT] RFF Classifier with a 300ms Prediction Window, which can be considered representative of all RFF classifiers. Whilst the classifier clearly identifies the Step Back movement, the ability of the classifier to differentiate between the Step Forward and Feet Together step types is reduced compared to the LFF classifier. This may be due to the PCA, which may reduce the dimensionality of the data in a direction which was used to differentiate between the two movements in the LFF classifier. Alternatively, the behaviour of the participant may have changed between the right and left sides.

Prediction outcome

		SF	FT	SB
Actual Value	SF'	0.79	0.22	0.00
	FT'	0.21	0.78	0.00
	SB'	0.00	0.00	1.00

Fig. 7. Average Confusion Matrix for the [CM+CF+MT] Right Foot Forward Classifiers, 300ms Prediction Window. SF = Step Forward, FT = Feet Together, SB = Step Back

#### B. Implications for Classifier Prediction Window

Practically, a shorter prediction window would be advantageous in terms of prediction time (reducing delay) and processor requirements. It was hypothesised that, after some threshold time period (*e.g.* the amount of time required for the user to physically prepare for the movement) there would be limited advantage in a longer prediction window. This appears to be true in the RFF classifier, with similar performance across all prediction windows in all sensor data combinations. In contrast, the LFF classifier's accuracy consistently improves with longer prediction windows by similar amounts. Thus, the results presented here do not present any notable or obvious candidates for an optimal prediction window, with such a decision likely to be more reliant on other requirements, such as definition of an acceptable processing delay or minimum accuracy requirement.

#### C. Implications for Sensor Set

The results unsurprisingly suggest that additional information provides the classifier with more information to make a better prediction. However, it is interesting to note the relative importance that additional measurements, particularly on the crutches, can provide. The LFF classifier saw relatively limited improvement in performance with the addition of the single axial crutch force — on average 4.5% across prediction windows – but a larger improvement with the addition of all forces 7.1%. In contrast, the RFF classifier saw its largest improvement with the addition of axial forces only (5.9%), with less improvement with additional sensors after that (3.4% and 2.0% for all forces and moments respectively). This suggests that the additional direction information about the forces at the end of the crutch may be useful for differentiating between movements close to each other, but potentially less useful for larger differences.

## D. Implications for Extension to More Movement Classes

The ultimate goal of this work was to apply the process developed here to a system with a larger variety of movement transitions. Whilst the performance of the LFF classifier is promising, given the performance level achieved and the similarity between movements, the RFF detracts from this success as more options are introduced. It is also noted that, with a smaller number of transitions it may be useful to bias the classifier towards one which has a larger-margin separating hyperplane — which may be achieved by using a smaller "C parameter" (at the cost of potentially reduced accuracy). For simplicity and for a fair comparison, the "C parameter" was not modified between classifiers. However, this may be a parameter which may require tuning to improve performance if more movement classes were considered.

## E. Limitations and Future Work

There are a number of limitations associated with this work. First, this work is a case study, with the classifiers created and evaluated with subsets of a limited data set captured from a single user, with no testing between individuals performed. Initial tests (not reported here) suggest that performance with data from other individuals is poor, which may mean that a unique classifier must be developed for each individual. This should be further investigated. Secondly, the user themselves selected the movements within the testing protocol. Implementation and testing of the final classifier design in a 'live' system, and evaluation of the performance, remains a task to be performed. Thirdly, the set of sensors used in this work is quite limited, with additional sensors potentially including inertial measurement units, or in-sole feet sensors potentially providing better performance. This seems likely given that including additional sensors categorically improved performance.

## VI. CONCLUSIONS

This work explored the effects of changing the input data to a classifier against the performance of said classifier in predicting user intent, with the view that such a classifier may be useful in a 'continuous prediction' paradigm. This study included only a single, able-bodied participant, and thus the results should be treated with caution, however, the results indicated that that approach may have promise, with potential advantages in increasing the available sensor set beyond that considered within this work.

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