

A DIALOGICAL ACCOUNT OF LOGICAL PLURALISM

ROHAN FRENCH

• rohan.french@gmail.com • Melbourne Logic Group • September 2016

1 THE EXPLANATORY PROBLEM FOR LOGICAL PLURALISM

- Logical pluralists hold that there are multiple different correct logics which we can use to assess the validity of any given argument, any of which may deliver inconsistent validity judgements.

A characterisation of a pluralist position needs to explain what it is to endorse all of the various consequence relations the pluralist accepts and how they relate to an intuitive notion of logical consequence. Keefe (2014, p.1376)

- The logical pluralist needs to explain two things: first, they need to explain how there can be more than one correct notion of logical consequence; and secondly, why these various different notions of logical consequence are notions of *logical* consequence.

2 THE BUILT-IN OPPONENT CONCEPTION OF DEDUCTION

- A dialogical, multi-agent conception of the nature logic and deductive proof introduced in Dutilh Novaes (2013, 2015).
- Deductive arguments correspond to a specialised kind of semi-adversarial dialogues—dialogical interactions with both an adversarial as well as a cooperative component. The two participants in these dialogues have opposed goals:
 - The PROVER (=Proponent) seeks to establish that a certain conclusion follows from given premises
 - The SKEPTIC (=Opponent) seeks to block the establishment of this conclusion, asking pointed ‘why does this follow?’ questions (i.e. ‘if you can show *this* i’ll accept *that*’)
- In essence the Skeptic asks for, and the Prover gives, reasons for their claims of logical consequence.

Proponent’s job is not only to ‘beat Opponent’; she also seeks to *persuade* Opponent of the truth of the conclusion, if he has granted the truth of the premises. In fact, the goal is not only to show *that* the conclusion follows from the premises, but also *why* it does; this corresponds to the idea that deductive arguments ought to have explanatory value. In this sense, Proponent and Opponent are cooperating in a common inquiry to establish what follows from the premises, and thus to further investigate the topic in question. Dutilh Novaes (2015, p.599f)

- Skeptic’s role also involves higher-order cooperation: he must say what would convince them, and not obstinately refuse to admit that the premises follow from the conclusion.
- The other important aspect of the present approach is the idea that over time Skeptic has been progressively

‘silenced’ and idealised, until they are no longer an active participant in a deductive dialogue but have instead become part of the deductive method itself. In essence their role has been INTERNALISED and is played ‘offline’ by Prover. In order to beat such an idealised opponent Prover needs to ensure that their inferential steps are indefeasible.

3 INTRODUCING PROVER-SKEPTIC GAMES

- *Prover-Skeptic games* are a kind of dialogue game introduced in (Sørensen and Urzyczyn, 2006, p.89–94) to game-theoretically characterise the implicational fragment of intuitionistic logic. We follow (and build on) the presentation of such games given in Shoham and Francez (2008).

*In the library two logic students, **Penelope** and **Scott** are arguing over the validity of the argument from $p \rightarrow q$ and $q \rightarrow r$ to $p \rightarrow r$. Penelope thinks that this argument is valid and is trying to convince Scott, who is skeptical.*

- **Penelope (1):** I reckon that $p \rightarrow r$ follows from $p \rightarrow q$ and $q \rightarrow r$
- **Scott (1):** Yeah? Well if that’s so then suppose I grant you $p \rightarrow q$ and $q \rightarrow r$ along with p , how are you meant to get r ?
- **Penelope (2):** If you grant me q I can get r from $q \rightarrow r$ (which you just granted).
- **Scott (2):** But why should I grant you q ?
- **Penelope (3):** Well if you were to grant me p then I could get q from $p \rightarrow q$ which you granted at the start.
- **Scott (3):** But why should I grant you p ?
- **Penelope (4):** Because you granted it to me at the start!
Penelope leaves the library triumphantly.

Figure 1: **A Dialogue** representing the kind of situation modelled by Prover-Skeptic games.

- We stick with the language \mathcal{L} of implicational logic, the formulas of which are constructed out of a countable supply of propositional variables p_0, p_1, p_2, \dots using the binary connective ‘ \rightarrow ’ of implication. Through we will be concerned with the natural deduction system in Figure 2.

AXIOMS

$$A \succ A$$

INTRODUCTION/ELIMINATION RULES

$$\frac{\Gamma, A \succ B}{\Gamma \succ A \rightarrow B} (\rightarrow I) \quad \frac{\Gamma \succ A \quad \Delta \succ A \rightarrow B}{\Gamma, \Delta \succ B} (\rightarrow E)$$

STRUCTURAL RULES

$$\frac{\Gamma, A, A \succ B}{\Gamma, A \succ B} (W) \quad \frac{\Gamma \succ B}{\Gamma, A \succ B} (K)$$

Figure 2: Our Sequent-to-Sequent Style Natural Deduction System

Definition 3.1. (Notation) Given a set of sequents S and sequents β, γ let us write:

- $\beta \Rightarrow_{(I)}^* \gamma$ whenever we can derive γ from β using zero or more applications of our introduction rules.
- $S \Rightarrow_{(E)}^* \beta$ iff we can derive β from the sequents in S via zero or more applications of our elimination and structural rules.

Definition 3.2. (Dialogue Game) A dialogue over (Γ, A) is a (possibly infinite) sequence $S_1, \beta_1, S_2, \beta_2, \dots$ where each S_i is a multiset of sequents, and each β_i a sequent where:

1. $S_1 = [\Gamma \succ A]$
2. $\beta_i \Rightarrow_{(I)}^* \sigma$ for some non-axiomatic $\sigma \in S_i$, where $\beta_i = \Gamma' \succ p_i$ for some p_i , and multiset Γ' .
3. $S_{i+1} \Rightarrow_{(E)}^* \beta_i$

- We can understand a *Skeptic-move* β as doubting σ and asking Prover to show β .
- Similarly we can understand a *Prover-move* S in response to a Skeptic-move β as Prover providing the grounds for their assertion that σ .
- We will say that a Prover *wins a dialogue* if we reach a point at which Skeptic can make no further move. Further, we have that if $\Gamma \succ A$ is provable in our natural deduction system, then Prover has a *winning strategy* in the dialogue over (Γ, A) .

4 INTERNALISATION & COOPERATION

- Skeptic's moves do not require any form of creativity to perform. Given that their role is essentially deterministic it is easy to see how it is apt to be performed 'offline' by a Prover who follows this method.
- We have the right mix of adversariality and cooperation between Prover and Skeptic present in Prover-Skeptic games. Adversariality results in Prover attempting to offer only derivable sequents. Cooperation seen in players

having to agree on what counts as a challenge, and an acceptable response to a challenge.

The adversarial component is what accounts for the property of necessary truth preservation; the cooperative component is what explains the ideal of perspicuity and explanatoriness. (Dutilh Novaes, 2015, p.599f)

- It is from the cooperative component that pluralism arises.

5 PROOF AND EXPLANATION

- Prover moves are meant to do more than merely show that the premises follow from the conclusion, they're meant to be *explanatory*.
- The notion of explanation which is most appropriate here is broadly 'pragmatic' in nature, thinking of explanation as answers to why-questions following Van Fraassen (1988).
- A why-question is just a request for information of a certain sort, and what information is requested can differ from context to context. (Van Fraassen, 1988, p.153).
- In the present context, the information requested is of the form 'why does this follow', with the answer intended to convince the Skeptic that it follows. But what counts as a satisfactory answer to such a question may vary from context to context. If it does in the right way, then we will have a naturally arising form of folk-relativism about logical consequence.

6 A ROUTE TO PLURALISM

- Do different constraints on what counts as an acceptable response to a Skeptic challenge have logical repercussions? One plausible kind of constraint that might arise concerns the level of detail, or granularity, of the required explanation. Indeed we can see this in mathematical practice, where different levels of detail are requested and required in teaching as opposed to research situations (Hersh, 1993).
- There do seem to be cases where changes in what counts as acceptable answer the question of 'why does this follow' have logical repercussions.

6.1 Foundations of Geometry

- In Pambuccian (2004) we are told that work in the foundations of geometry at the beginning of the 20th century appears to display a concern, not just for what axioms are used in proving a particular theorem, but also in the *number of times* that such axioms are, or indeed must, be so appealed to.
- For example, G. Hessenberg in 1905 proves that Desargues axiom follows from a collection of axioms for plane projective geometry along with a threefold use of the Pappus axiom. Pambuccian notes that all known proofs both require, and explicitly mention, this three-

fold use.

- So this can be seen as a situation in which what counts as an acceptable answer to the question of ‘why does this follow’ must not just display that the challenged claim follows ‘of necessity’ from the claims provided, but also the extent to which different claims are used in showing this. This ‘resource consciousness’ suggests an abandoning of the structural rule of contraction, according to which whenever A follows from Γ and two copies of B, and when A follows from Γ and a single copy of B.

6.2 Confirmation

- According to Glymour (1980) Hypothetico-Deductivism is “hopeless as a method of confirmation because it cannot maintain the condition of relevance between a hypothesis and the evidence which confirms it”, as Waters (1987, p.453) summarises the main thrust of Glymour’s very short paper.
- Attempts to save the Hypothetico-Deductive model of confirmation from Glymour’s charges (e.g. those in Schurz (1991), Gemes (1993)) all appear to put forward constraints which are variations on the theme of Relevant logic à la Anderson and Belnap (1975).
- This suggests that in contexts where we are concerned with confirmation the norms on explanation require the appropriate kind of connection of relevance between theory, hypothesis and evidence when we are trying to show that the theory and hypothesis entail the evidence. At the very least this suggests a rejection of the structural rule of weakening.
- These cases appear to give us places where the norms on explanation which govern deductive dialogues appear to be (contra (Beall and Restall, 2006, p.88)) relative to communities of inquiry, or perhaps better, relative to the particular interests of those communities.

7 PROVER-SKEPTIC GAMES FOR SUBSTRUCTURAL LOGICS

- We can model this in our Prover-Skeptic games in the obvious way, by placing constraints on which structural rules can be appealed to in a Prover move.
- So, for example, in the context of concerns about the foundations of geometry we cannot use contraction in our Prover moves (resulting in the games characterising affine or linear logic), and in confirmation contexts we cannot use weakening (resulting in the games characterising relevant logic).

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