CONFERENCE on NONLINEAR WAVES and MATHEMATICAL GENERAL RELATIVITY

Tsinghua University

July 10 to July 14, 2023

Conference Schedule

Monday	Speaker	Title
8:00-8:30	Sign-Up	
8:30-8:40	Opening	
8:40-9:30	Sergiu Klainerman	The nonlinear stability of slowly rotating Kerr
9:40-10:30	Piotr Chrusciel	Gluing variations
Tea Break		
11:00-11:50	Pieter Blue	Linear stability of the Kerr spacetime in the outgoing radiation gauge
Lunch		
14:00-14:50	Yi Zhou	New div-curl lemma and applications to non-linear wave and Schrodinger equations
15:00-15:50	Siyuan Ma	Price's law and Strong Cosmic Censorship for linearized fields in Kerr spacetimes
Tea Break		
16:20-17:10	Tianwen Luo	On multi-dimensional rarefaction waves

Tuesday	Speaker	Title
8:30-9:20	Qian Wang	Rough solutions of the 3-D compressible Euler equations
9:30-10:20	Xinliang An	Naked Singularity Censoring in General Relativity
Tea Break		
10:50-11:40	Li Xu	Long time existence for surface waves Boussinesq systems
Lunch		
14:00-14:50	Xiao Zhang	Spherically symmetric Einstein-scalar-field equations for wave-like decaying null infinity
15:00-15:50	Chengbo Wang	Blow-up for semilinear wave equations on Kerr black hole backgrounds
Tea Break		
16:20-17:10	Junbin Li	On the instability of naked singularities in general relativity

Wednesday	Speaker	Title
8:30-9:20	Joachim Krieger	TBA
9:30-10:20	Dong Li	Turbulent solutions for 2D quasilinear wave
Tea Break		
10:50-11:40	Arick Shao	Bulk-boundary correspondence for vacuum asymptotically Anti-de Sitter space-times
Lunch		
14:00-14:50	Volker Schlue	Scattering for wave equations with sources in the wave zone
15:00-15:50	Haoyang Chen	Low regularity ill-posedness for elastic waves and for MHD system in 3D and 2D
Tea Break		
16:20-17:10	Lili He	The linear stability of weakly charged and slowly rotating Kerr-Newman family of charged black holes

Thursday	Free Discussions	
Friday	**Speaker**	**Title**
8:30-9:20	Yue Ma	Wave-Klein-Gordon systems in 1+2 dimension-al space-time with nonlinear interactions in di-vergence form
9:30-10:20	Yang Lan	Strongly interacting multi-soliton for general-ized Benjamin-Ono equations
Tea Break		
10:50-11:40	Guixiang Xu	Minimal mass blow-up solutions for the L^2-critical NLS with the Delta potential in one dimension
Lunch		
Free discussions in the afternoon		

Title and Abstract

Xinliang An (National University of Singapore)

Title: Naked Singularity Censoring in General Relativity

Abstract: In this talk, I will report several results related to naked singularity censoring and weak cosmic censorship.

Pieter Blue (University of Edinburgh)

Title: Linear stability of the Kerr spacetime in the outgoing radiation gauge

Abstract: This talk will discuss a new gauge condition (i.e. coordinate condition) for the Einstein equation, the linearisation of the Einstein equation in this gauge, and the decay of solutions to the linearised Einstein equation around Kerr black holes in this gauge. The stability of the family of Kerr black holes under the evolution generated by the Einstein equation is a long-standing problem in mathematical relativity. In 1972, Teukolsky discovered equations governing certain components of the linearised curvature that are invariant under linearised gauge transformations. In 1975, Chrzanowski introduced the "outgoing radiation gauge", a condition on the linearised metric that allows for the construction of the linearised metric from the linearised curvature. In 2019, we proved decay for the metric constructed using Chrzanowski's outgoing radiation gauge. Recently, using a flow along null geodesics, we have constructed a new gauge such that, in this gauge, the Einstein equation is well posed and such that the linearisation is Chrzanowski's outgoing radiation gauge. This is joint work with Lars Andersson, Thomas Backdahl, and Siyuan Ma.

Haoyang Chen (National University of Singapore)

Title: Low regularity ill-posedness for elastic waves and for MHD system in 3D and 2D

Abstract: We construct counterexamples to the local existence of low-regularity solutions to elastic wave equations and to the ideal compressible magnetohydrodynamics (MHD) system in three and two spatial dimensions (3D and 2D). For 3D, inspired by

the recent works of Christodoulou, we generalize Lindblad's classic results on the scalar wave equation by showing that the Cauchy problems for 3D elastic waves and for 3D MHD system are ill-posed in H^3 and H^2, respectively. Both elastic waves and MHD are physical systems with multiple wave-speeds. We further prove that the ill-posedness is caused by instantaneous shock formation, which is characterized by the vanishing of the inverse foliation density. In particular, when the magnetic field is absent in MHD, we also provide a desired low-regularity ill-posedness result for the 3D compressible Euler equations, and it is sharp with respect to the regularity of the fluid velocity. Our proofs for elastic waves and for MHD are based on a coalition of a carefully designed algebraic approach and a geometric approach. In 2D, we prove the $H^11/4$ and $H^7/4$ ill-posedness for the elastic wave equations and ideal MHD system (also for Euler equations). Compared with the 3D case, the construction of ill-posed profile in 2D is more delicate. This talk is based on joint works with Xinliang An and Silu Yin.

Piotr Chrusciel (University of Vienna)

Title: Gluing variations

Abstract: I will review various gluing/extension/embedding constructions for Einstein equations.

Lili He (Johns Hopkins University/Princeton University)

Title: The linear stability of weakly charged and slowly rotating Kerr-Newman family of charged black holes

Abstract: I will discuss the linear stability of weakly charged and slowly rotating Kerr-Newman black holes under coupled gravitational and electromagnetic perturbations. We show that the solutions to the linearized Einstein-Maxwell equations decay at an inverse polynomial rate to a linearized Kerr-Newman solution plus a pure gauge term. The proof uses tools from microlocal analysis and a detailed description of the resolvent of the Fourier transformed linearized Einstein-Maxwell operator at low frequencies.

Sergiu Klainerman (Princeton University)

Title: The nonlinear stability of slowly rotating Kerr

Joachim Krieger (École Polytechnique Fédérale de Lausanne)

Title: TBA

Yang Lan (Tsinghua University)

Title: Strongly interacting multi-soliton for generalized Benjamin-Ono equations

Abstract: We consider the generalized Benjamin-Ono equation: $\partial_t u-(|D|u-|u|^{p-1}u)_x=0$ with L^2-supercritical power p>3 or L^2-subcritical power 2<p<3. We will construct strongly interacting multi-solitary wave of the form: $\sum_{i=1}^n Q(\cdot-t-x_i (t))$, where the parameters x_i(t) satisfying $x_i(t)-x_{i+1}(t)\sim \sqrt{t}$ as t goes to infinity. We will also prove the uniqueness of such solutions in the case of n=2 and p>3.

Dong Li (Hongkong University)

Title: Turbulent solutions for 2D quasilinear wave

Abstract: I will discuss a number of recent results in connection with Alinach's blowup-at-infinity problem. Time permitting I will explain some recent progress on the non-compact case.

Junbin Li (Sun Yat-sen University)

Title: On the instability of naked singularities in general relativity

Abstract: The weak cosmic censorship in general relativity states that naked singularities cannot appear in gravitational collapse generically. In this talk, I will report some research progress on the instability of naked singularities.

Tianwen Luo (South China Normal University)

Title: On multi-dimensional rarefaction waves

Abstract: We study the two-dimensional acoustical rarefaction waves under the irrotational assumptions. We provide a new energy estimates without loss of derivatives. We also give a detailed geometric description of the rarefaction wave fronts. As an

application, we show that the Riemann problem is structurally stable in the regime of two families of rarefaction waves. This is a joint work with Pin Yu.

Siyuan Ma (Chinese Academy of Sciences)

Title: Price's law and Strong Cosmic Censorship for linearized fields in Kerr space-times

Abstract: I will show the precise late-time asymptotics for linearized fields globally in Kerr exterior and interior regions, which are called the Price's law and closely relevant to the Strong Cosmic Censorship, respectively. This is joint work with Lin Zhang.

Yue Ma (Xi'an Jiaotong University)

Title: Wave-Klein-Gordon systems in 1+2 dimensional spacetime with nonlinear interactions in divergence form

Abstract: In this talk I will regard the global wellposedness of a type of systems composed by nonlinear wave and Klein-Gordon equations in 1+2 dimensional spacetime with small and regular initial data. This type of systems can be found in differential geometry and plasma physics. The divergence structure of the Klein-Gordon-wave interaction is the key for global existence in this lower-dimensional case. These are joint works with S. Duan and W. Zhang.

Arick Shao (Queen Mary University of London)

Title: Bulk-boundary correspondence for vacuum asymptotically Anti-de Sitter spacetimes

Abstract: The AdS/CFT conjecture in physics posits the existence of a correspondence between gravitational theories in asymptotically Anti-de Sitter (aAdS) spacetimes and field theories on their conformal boundary. In this presentation, we prove a rigorous mathematical statement toward this conjecture in the classical relativistic setting. In particular, we show there is a one-to-one correspondence between AdS solutions of the Einstein-vacuum equations and a suitable space of data on the conformal boundary (consisting of the boundary metric and the boundary

stress-energy tensor), provided the boundary satisfies a geometric condition. We also discuss applications of this result to symmetry extension, as well as its connection to unique continuation problems. This is joint work with Gustav Holzegel, and builds upon joint works with Athanasios Chatzikaleas, Simon Guisset, and Alex McGill.

Volker Schlue (The University of Melbourne)

Title: Scattering for wave equations with sources in the wave zone

Abstract: For non-linear wave equations satisfying the null condition or weak null condition the scattering problem from infinity is non-trivial due to slow interior decay, and the presence of logarithmic terms in the asymptotic expansion. In many cases the problem reduces to a scattering problem for wave equations with sources concentrated on the lightcone, which we solve with a systematic study of the homogeneous solutions of degree -1 and -2 in the interior, and exterior of the lightcone. Thus we construct scattering solutions with non-decaying radiation fields, which satisfy novel matching conditions to homogeneous asymptotics in the exterior and interior. The proof requires a delicate analysis of the fundamental solution, and invokes the Funk transform. This talk gives an overview of my recent results obtained jointly with Hans Lindblad.

Chengbo Wang (Zhejiang University)

Title: Blow-up for semilinear wave equations on Kerr black hole backgrounds

Abstract: In this talk, we will examine solutions to semilinear wave equations on black hole backgrounds and give a proof of an analog of the blow up part of the John theorem, with $F_p(u)=|u|^{p}$, on the Schwarzschild and Kerr black hole backgrounds. Our approach is also robust enough to be applied to focusing nonlinearity $F_p(u)=|u|^{p-1}u$ for the Schwarzschild backgrounds. It is a joint work with Mengyun Liu.

Qian Wang (University of Oxford)

Title: Rough solutions of the 3-D compressible Euler equations

Abstract: I will talk about my work on the compressible Euler equations. We prove

the local-in-time existence the solution of the compressible Euler equations in 3-D, for the Cauchy data of the velocity, density and vorticity $(v,\varrho, \omega) \in H^s\times H^s\times H^{s'}$, $2<s'<s$. The result extends the sharp result of Smith-Tataru and Wang, established in the irrotational case, i.e $\omega=0$, which is known to be optimal for $s>2$. At the opposite extreme, in the incompressible case, i.e. with a constant density, the result is known to hold for $\omega\in H^s$, $s>3/2$ and fails for $s\le 3/2$, see the work of Bourgain-Li. It is thus natural to conjecture that the optimal result should be $(v,\varrho, \omega) \in H^s\times H^s\times H^{s'}$, $s>2, \, s'>\frac{3}{2}$. We view our work as an important step in proving the conjecture. The main difficulty in establishing sharp well-posedness results for general compressible Euler flow is due to the highly nontrivial interaction between the sound waves, governed by quasilinear wave equations, and vorticity which is transported by the flow. To overcome this difficulty, we separate the dispersive part of sound wave from the transported part, and gain regularity significantly by exploiting the nonlinear structure of the system and the geometric structures of the acoustic spacetime.

Guixiang Xu (Beijing Normal University)

Title: Minimal mass blow-up solutions for the L^2-critical NLS with the Delta potential in one dimension

Abstract: Local well-posedness theory together with sharp Gagliardo-Nirenberg inequality and the conservation laws of mass and energy implies that the solution with mass less than $\|Q\|_{2}$ is global existence in $H^1(\R)$, where Q is the ground state of the L^2-critical NLS without the delta potential. We are interested in the dynamics of the solution with threshold mass $\|u_0\|_{2}=\|Q\|_{2}$ in $H^1(\R)$. First, for the case $\mu=0$, such blow-up solution exists due to the pseudo-conformal symmetry of the equation, and is unique up to the symmetries of the equation in $H^1(\R)$ by F. Merle and recently in $L^2(\R)$ by B. Dodson. Second, for the case $\mu<0$, simple variational argument with the conservation laws of mass and energy implies that radial solutions with threshold mass exist globally in $H^1(\R)$. Last, for

the case $\mu>0$, we show the existence of radial threshold solutions with blow-up speed determined by the sign (i.e. $\mu>0$) of the delta potential perturbation since the refined blow-up profile to the rescaled equation is stable in a precise sense. The key ingredients here including the Energy-Morawetz argument and compactness method as well as the modulation analysis, which are close to Raphael-Szeftel's argument for inhomogeneous case. This is a joint work with Xingdong Tang (UIST, Nanjing).

Li Xu (Beihang University)

Title: Long time existence for surface waves Boussinesq systems

Abstract: We prove long time existence results for the solutions of Boussinesq system modeling the propagation of long, weakly nonlinear water waves. The results contain all cases of linearly well-posed system in the (abcd) family of Boussinesq systems. This is a joint work with Jean-Claude Saut and Chao Wang.

Xiao Zhang (Chinese Academy of Sciences / Guangxi University)

Title: Spherically symmetric Einstein-scalar-field equations for wave-like decaying null infinity

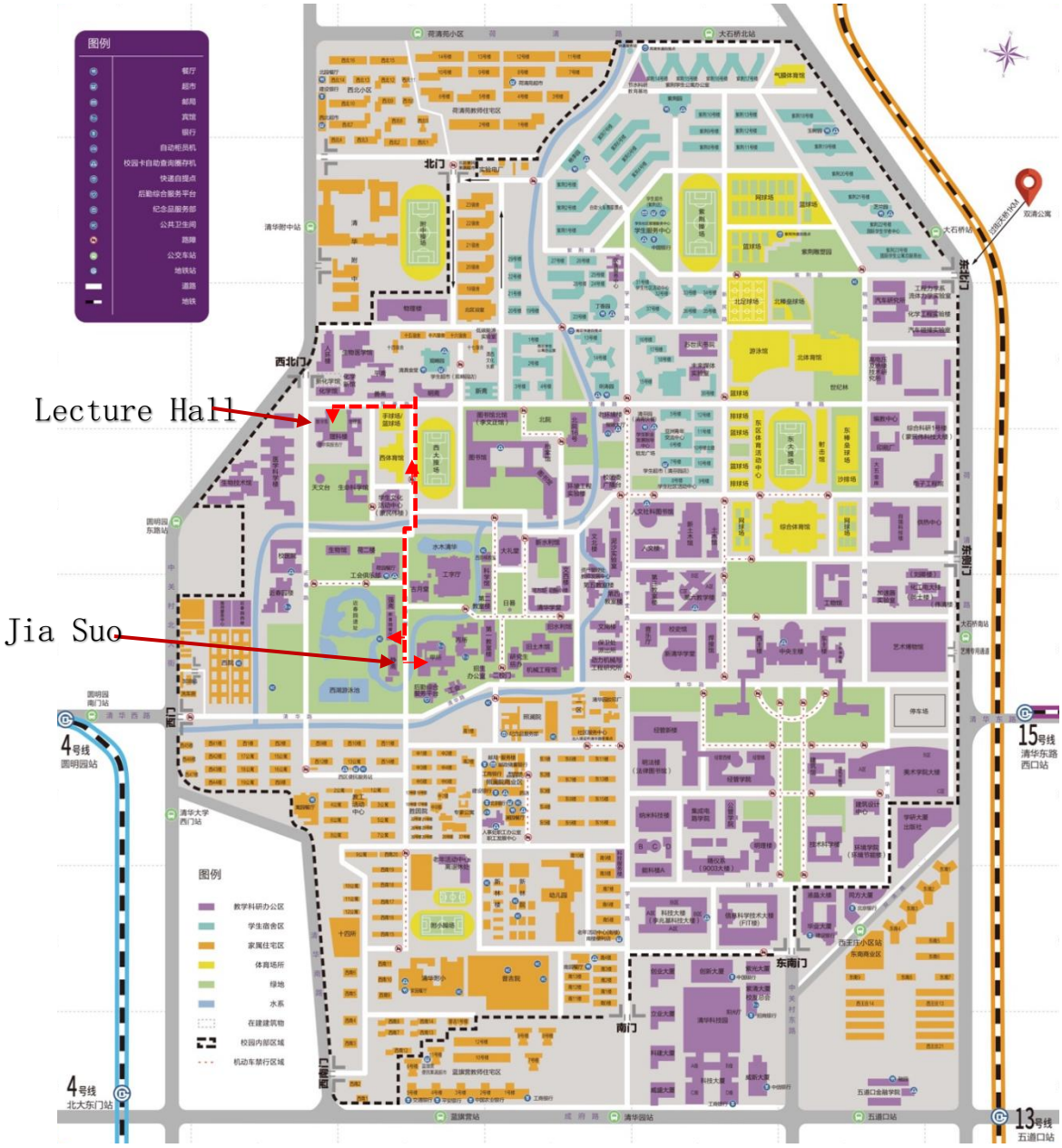
Abstract: Spherically symmetric Einstein-scalar-field equations were studied systematically by Christodoulou since 1986. In particular, he proved that unique global solutions exist for small "particle-like" decaying initial data at null infinity. In this talk, we show that unique global solutions exist for small "wave-like" decaying initial data at null infinity. We also generalize this result to more general case where potentials are nontrivial. The talk is based on the joint works with Chuxiao Liu.

Yi Zhou (Fudan University)

Title: New div-curl lemma and applications to nonlinear wave and Schrodinger equations.

Abstract: New studies of the wave and Schrodinger equations by the speaker seem to reveal that the momentum balance law is equally important than the energy balance law, even more important than the energy balance law. The advantage of the energy

balance law is obvious because it gives the energy which is coercive. However, we discover that there are also ways to obtain coercive estimates from the momentum balance law. Our new div-curl lemma is one way of achieving this goal. I will explain how to do this and its applications to nonlinear wave and Schrodinger equations, including the Schrodinger map flow in one space dimensions and radial minimal surface equation in 1+3 dimensional Minkowski space.



Please take me to Jia Suo of Tsinghua University.

请带我去清华大学甲所，谢谢！

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